A Numerical Revolution: Mathematical innovation and the growth of pre-modern European economies

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Abstract

The accumulation of knowledge and its application to human needs are discontinuous processes leading to economic growth through innovation and change. In this paper, we focus on the diffusion of mathematical innovation in pre-modern Europe (13th–16th century). Using an original dataset of over 1000 manuals of practical arithmetic, we produce historiographical and econometric evidence of the economic importance of the switch from Roman to Indo-Arabic numerals (0 to 9) among Western European economic actors. We show that, by laying the foundations for new business practices observed from the 13th century onwards, the spread of this new numeral system had a positive and significant effect on the growth of European cities. The key mechanism through which the growth potential of the new numbers was realized was human capital formation—i.e. the training of the merchant class—and its effects precede those of the printing press.

JEL codes: O3; O4; N13; N3

Keywords: Human capital; knowledge diffusion; learning; economic growth; general-purpure technologies

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1 Introduction

Until the late Middle Ages, people in Europe relied on tallies, finger reckoning and Roman numerals to represent numbers, and on reckoning boards to carry out calculations. While these systems were effective for calculating addition and subtraction with natural numbers, they were cumbersome for multiplication and division, and for handling large and rational numbers. By overcoming the limitations of traditional European systems, the adoption of Indo-Arabic numerals[1] in European commercial practices, which began in Southern Europe in the 13th century, enabled new applications of mathematics to practical economic problems.

Thanks to the principle of place value and to the symbol for zero, Indo-Arabic numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) provided a system that was at the same time a reckoning tool and a notation technique to record numbers. These distinctive characteristics, in fact, make it possible to use a limited set of symbols (i.e. just ten figures) to both represent numbers and make algorithmic calculations with them (Ifrah 2000, 679). Moreover, as Indo-Arabic numerals were introduced into Europe together with the notation for fractions, they made it possible to represent and to calculate with virtually *any* rational number. It is indeed significant that the first European economic agents to adopt Indo-Arabic numerals were also the first to develop new business practices that required accurate calculations with rational numbers. These included the bill of exchange, modern insurance contracts, and the management of international businesses based on the distribution of profits and losses according to shares—all characteristic features of the 'commercial revolution' of the 13th century, and fundamental stepping stones in the development of modern market institutions (De Roover 1953).

In this paper, we study the relationship between the diffusion of Indo-Arabic numerals and the growth of pre-modern European economies. Building on previous work on the determinants of European long-run growth (De Long and Shleifer 1993; Acemoglu et al. 2005;

1. We adopt the term *Indo-Arabic numerals* to avoid the potential religious connotations of the expression *Hindu-Arabic numerals*, following Daston (2022, 289).

Buringh and Van Zanden 2009; Dittmar 2011; De Pleijt and Van Zanden 2016; Curtis and De la Croix 2026), we show that the late medieval and early modern period was characterized by an underappreciated process of growth through the accumulation of mathematical knowledge and its applications to commerce. We proxy the spread of these techniques by focusing on the diffusion of practical arithmetic over the period 1300-1600, and we explore its importance over the long run. We combine historiographical evidence with econometric analyses of the relationship between the diffusion of Indo-Arabic numerals, city-level growth, and other indicators of economic development. The study is based on a new and original dataset of over a thousand manuals of practical arithmetic. The dataset was constructed from existing catalogs and extensive archival research, and stretches back well before the introduction of the printing press.

The paper expands upon the broader theme concerning the role of knowledge in the process of economic growth (Aghion and Howitt 1992; Romer 1990; Grossman and Helpman 1993; Jones 1995; Weitzman 1998). The kind of new knowledge we are dealing with entails a codified component—it is contained and preserved through time in manuscript and printed texts—as well as a tacit component (Polanyi 1966; Dasgupta and David 1994) that resided in and was developed by the people who adapted and transmitted this knowledge as a foundation for advanced commercial skills.

A stream of studies has documented the fundamental role played by advanced human capital in the early modern period, exploring both the contribution of practically-minded intellectuals and that of socially humbler figures, such as artisans, workers, and other practitioners—what Mokyr has described as the emergence of 'useful knowledge' (Berg 1994; Mokyr 2002; Berg 2007; Hilaire-Pérez 2007; Allen 2009; Squicciarini and Voigtländer 2015; Mokyr 2017). Kelly, Mokyr, and Ó Gráda have stressed the crucial complementarity between the applications of scientific knowledge to practical problems and the dexterity of the labor force in the run-up to the British industrial revolution (Kelly et al. 2014, 2023). Kelly and Ó Gráda have recently argued that early modern 'mathematical practitioners' were key actors in these phenomena, showing that practical arithmetic is a crucial field to document the relationship between the accumulation of knowledge and economic growth (Kelly and Ó Gráda 2022). By providing a proxy for the spread of Indo-Arabic numerals among European practitioners, the dataset of practical arithmetic manuals allows us to reconstruct the increasing application of new mathematical knowledge. Over time, Indo-Arabic numerals influenced economic activity in a manner comparable to a general-purpose technology, enhancing a wide range of numerical activities and arguably generating slow long-term spillovers across fields including surveying, navigation, instrument making, engineering, shipbuilding, architecture, and ballistics (Danna 2026).

After presenting the historiographical framework, we first provide systematic descriptive results by means of pooled and panel OLS regressions with as good a set of controls as can be applied in this empirical context. Secondly, we employ a staggered difference-in-differences analysis to explore the possible causal effects of this 'numerical revolution' on the growth of European cities, the underlying mechanism being the training of the mercantile class.

2 Literature review and historical framework

2.1 Indo-Arabic numerals as mathematical innovation

There is a growing interest in investigating the role played by 'useful knowledge' and skills in the pre-industrial period (Mokyr 2017; Mokyr et al. 2022). Van Zanden and others started to explore this theme from a variety of perspectives, suggesting that 'human capital' played a relevant role in European growth since the late Middle Ages (Baten and Van Zanden 2008; Van Zanden 2009). Buringh and Van Zanden (2009) estimated that Europe developed a consistently growing production of texts—both manuscript and printed—since the Middle Ages, raising the possibility that Europe developed a 'knowledge economy' starting from that period. Epstein (2013) has stressed the importance of technical change and the transmission of technical knowledge in premodern Europe. De Pleijt and Van Zanden (2016) have argued that 'human capital formation' was 'the driver of growth' in the pre-industrial period, and that diverging economic outcomes of northern European areas with respect to southern and central Europe (the so-called 'little divergence') can be accounted for based on the accumulation of 'human capital'.

In an influential paper, Dittmar (2011) investigated the role of the printing press in the process of European growth, arguing that the introduction of this new general-purpose (information) technology had a causal effect on the growth of European cities. Other studies considered the role of specific institutions in fostering the development of markets, including those that secured the transmission of knowledge outside closed kinship systems, such as apprenticeships (De la Croix et al. 2018), and the role of universities (Cantoni and Yuchtman 2014; Curtis and De la Croix 2026). We focus on the spread of advanced commercial skills since the onset of the so-called 'commercial revolution' of the late Middle Ages. This revolution was a turning point in European economic history, as it marked the beginning of European economic expansion after the collapse of the Western Roman Empire (Lopez 1976). De Roover argued that the wave of financial and organizational innovations developed during this period was a 'complete or drastic change' in European commercial practices, comparable in scope to the industrial revolution (De Roover 1953).

During the commercial revolution, the first international commercial-banking companies appeared, based on a new kind of partnership contract—the *compagnia* (Goldthwaite 2009; Tognetti 2015; Padgett and McLean 2006). To manage these complex organizations, international merchant-bankers invented the foundational principle of modern accounting, i.e. double-entry bookkeeping (De Roover 1956; Melis 1972; Goldthwaite 2015). As they relied heavily on long-distance trade, these merchants also introduced the first insurance contracts (Ceccarelli 2020). Thanks to an accounting system integrated across different cities, these merchants were able to make international transfers by means of interlocking accounting operations registered in the ledgers of two branches. This is the origin of the bill of exchange, a fundamental financial innovation that made it possible to both move money internationally without moving specie, and extend short-term credit under the guise of a double operation of exchange (De Roover 1953; Lane and Mueller 1985; Bell et al. 2017). While the amount of precious metal shipped across the main European trading centers did not significantly change, the bill of exchange made it possible to drastically expand the monetary supply (Spufford 1988, 254–55; Lopez 1976, 72; Bolton and Guidi-Bruscoli 2021).

These developments characterized European business practices over the long run.[2] These commercial practices required little capital investment apart from the training necessary to handle them, and studying the mathematical techniques that made them possible is a highly appropriate angle to address the broader question of the role of knowledge as an engine of growth in pre-industrial Europe. The adoption of Indo-Arabic numerals was indeed a key enabling technology for the development of these commercial innovations. The positional numeral system reached Europe relatively late, i.e. in the late Middle Ages (Katz et al. 2016; Folkerts and Kunitzsch 1997; Folkerts 2001). Together with developing international merchant-banking, double-entry bookkeeping, insurance contracts, and the bill of exchange, the merchants of the commercial revolution were also the first European economic agents to adopt the positional numeral system (Van Egmond 1976; Danna 2026).

This was a major advancement over Roman numerals and other numerical tools used in Europe. Before the introduction of Indo-Arabic numerals, Western Europeans used different systems to calculate and to represent numbers. They used counter abacuses and multiplication tables to calculate, and tallies, finger reckoning and Roman numerals to represent numbers (Otis 2024; Chrisomalis 2010; Ifrah 2000). Thanks to the concept of place value and to the symbol for zero, Indo-Arabic numerals made it possible to use an efficient set of symbols (i.e. only ten figures) to both represent any number and to make algorithmic calculations with them, providing a numeral system that is at the same time a reckoning tool and a notation to represent any number (Ifrah 2000, 679).[3] High precision in calculation was

2. The bill of exchange remained the most important means for international exchange well into the 19th century; insurance contracts evolved until reaching today's modern form; and double-entry bookkeeping is still today practiced following the principles developed in the 13th and 14th centuries.

3. Note that European traditional numerical tools were effective for calculating and representing relatively small natural numbers, but they faced significant limitations in representing and manipulating large and rational numbers. Western Europeans, in fact, lacked a numerical notation that allowed them to represent

essential for the advanced business practices of the commercial revolution. For example, the continuous changes in the relative value of gold and silver coins made accountants handle complex calculations of exchange to convert values even in transactions that occurred within the same marketplace.[4] A high degree of numerical precision was required to issue bills of exchange, as merchants-bankers had to combine the calculations for conversions between local units with exchange rates across cities, a process that often implied dealing with complex rational numbers.[5] To use bills of exchange as instruments of credit meant combining these calculations with mastering the slightest differences between exchange rates, and primary sources provide ample evidence that these merchants actually performed similar calculations. Exchange rates in bills of exchange and account books were recorded in terms of a certain amount and *fraction* thereof of currency x for currency y (Bolton and Guidi-Bruscoli 2021).[6] One can wonder how (and to what extent) it would have been possible to develop similar commercial practices in the absence of a numerical notation that allowed for the handling of any rational number. Indo-Arabic numerals provided essential mathematical tools for the innovations of the commercial revolution. Moreover, by gradually spreading to sectors beyond commerce, they also enabled high-precision calculation in a broader set of practices.

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any rational number, because the Roman numerical system was developed to represent the duodecimal subdivisions of Roman units, and Roman 'fractions' had the structural limitation of representing only rational numbers based on the factors of twelve (Yeldham 1927; Maher and Makowski 2001). Conversely, thanks to the notation for fractions we still use today—which was adopted from the Arabic world together with the ten figures—Indo-Arabic numerals made it possible to represent and to operate with virtually any rational number. A numerical system that combined a reckoning tool and a notation to represent any number made it possible to enhance the precision of calculation significantly.

4. For instance, in Venice in the late 14th century, 1 *lira di grossi a monete* was valued 10 *ducati a monete*, and 1 *ducato a monete* was valued 64 *soldi di piccoli*. To calculate the value of 78 *lire di grossi a monete*, the brothers Tommaso and Zaccaria Talenti calculated (1 *lira* equalled 20 *soldi*): $(78 \times 10 \times 64)/20$, which yields 2496 *lire di piccoli*—the figure that appears in their accounts (Lane and Mueller 1985, 338).

5. Lane and Mueller (1985, 295) reconstructed that, between 1311 and 1330, a Bezant of Barbary varied between 13.6 and 13.9 Venetian *soldi a grossi*, a *perpero* of Costantinople varied between 26 and 32.8 Venetian *soldi a grossi*, and a *saracenato* of Costantinople between 30.33 and 45.5 Venetian *soldi a grossi*.

6. Similar quantitative problems arose in the calculations related to insurance contracts and to the establishment of commercial *compagnie* (consider the distribution of profits and losses according to shares). In commercial companies, partners usually received shares of profits according to quotas specified as *fractions*, or as a certain number of *soldi* and *denari* (and also *fractions* thereof) of a *lira* (the standard monetary unit of medieval Europe, equivalent to 20 *soldi*, with a *soldo* equivalent to 12 *denari*). In the 16th century, these shares could be expressed in such sophisticated terms as 3 *soldi*, 5 *denari*, and 17/23 of *denaro* for a specific partner (Goldthwaite 2015, 634).

2.2 Fibonacci's *Liber abaci*, practical arithmetic manuals and the 'abacus' schools

Leonardo Pisano's (better known as Fibonacci) *Liber abaci* provides early evidence of the knowledge exchanges between Italian maritime city-states and the western-Arabic world that led 13th-century merchants from these cities to become the first European economic agents to adopt Indo-Arabic numerals.[7] First completed in 1202, the *Liber abaci* is an advanced mathematical text that provides a synthesis of the Arabic and the European mathematical traditions, which cannot be considered as a text exclusively addressed to merchants (Giusti and D'Alessandro 2020). However, together with advanced mathematics, the *Liber abaci* also deals with the fundamentals of arithmetic, as it introduces the positional numeral system and the algorithms necessary to calculate with the ten figures. Moreover, it explicitly applies these mathematical tools to the solution of practical and commercial problems. As such, the *Liber abaci* documents the transfer of mathematical knowledge between mercantile communities that occurred across the shores of the Mediterranean in the first half of the 13th century (Giusti and Petti 2002).

While there is no direct evidence of the first use of Indo-Arabic numerals for economic purposes, they surely had been adopted by the end of the 13th century. During this period, Italian city-states developed a system of vocational schools. Among these schools, there were the so-called 'abacus schools', which secured the intergenerational transmission of practical

7. An interesting question concerns the training and business practices of merchants active in the late medieval Islamicate world. There is scant evidence on this, but there seem to have been no educational institutions comparable to the abacus school, the most prominent educational institution being the *madrasa*. *Madrasas* provided a form of higher education primarily aimed at training an elite of scholars, religious leaders, and high civil servants, more comparable to Western European universities (Fındıklı 2022), where mathematics was probably not aimed at commercial or other practical applications. Drawing from his comprehensive examination of the Cairo Geniza—a unique hoard of documents kept in the storeroom of the Ben Ezra Synagogue in Old Cairo—Goitein noted that in the documents pertaining to primary education "references to the study of Arabic calligraphy by children are frequent, but those on the teaching of arithmetic are scarce" (Goitein 1971, 177). Goitein questioned the hypothesis that merchants active in late medieval Cairo used Indo-Arabic numerals at all, and remarked: "we are usually oblivious of the fact that although the Arabic numerals had been in use in scientific works since early Islamic times, the merchants and other people engaged in economic life in the Arab world continued to use the entirely impractical Coptic (i.e., originally Greek) signs for numbers right down to modern times" (178).

arithmetic skills (Black 2007; Grendler 1989). Municipal records, chronicles, school curricula and personal *ricordanze* (i.e. memoirs) of former students indicate that these were lay and vernacular schools, where pupils between the ages of ten and fifteen were trained to use Indo-Arabic numerals, learned how to make calculations with them, and how to apply these mathematical techniques to solve practical problems. Although there were some regional differences (especially between Tuscan and Venetian cities), the curricula of these schools did not differ significantly in mathematical subjects (Grendler 1989; Goldthwaite 1972; Ulivi 2008).

These schools were founded in most central-northern Italian cities, and there is evidence of their presence also in some centers in the south. Archival evidence shows that the fees to attend these schools were low enough to make it possible also for children of small shopkeepers to afford a training in abacus mathematics (Van Egmond 1977; Goldthwaite 1972). While most pupils were prospective merchants, abacus schools were also attended by a broad spectrum of practitioners, who acquired foundational mathematical training there *before* continuing their education through apprenticeship (Black 2007; Grendler 1989; Goldthwaite 1972). Abacus masters were mathematical practitioners who belonged to a sort of urban middle class and were also employed as expert reckoners, for example in surveying and in auditing accounts for both the city state and for private citizens (Ulivi 2002; Van Egmond 1977; Goldthwaite 1972).[8]

The most important documentation we have from these schools is practical arithmetic manuals. These texts were mostly written in vernacular, i.e. the working language of merchants and practitioners. As the vast majority of the known authors of these manuals were abacus masters, it is reasonable to think that these texts were mainly used within abacus schools. The structure of these texts is also coherent with this hypothesis, as abacus manuals mainly consist of long lists of worked examples, organized into sections according to their domain of

8. Giovanni di Bicci de' Medici, the founder of the Medici bank, consulted abacus masters to learn about new algebraic methods for unsolved forms of quadratic and cubic equations, which were crucial to calculate compound interest rates (Ulivi 2015).

application (e.g. problems of conversion, of exchange, of division of profits and losses, etc.) (Bocchi 2017, 10–65; Franci 2015).

These texts had a wide social circulation. Estimates on the average enrollment of pupils in abacus schools range from 25 to 50 students (Grendler 1989, 72; Van Egmond 1976, 105–13).[9] Even by using the lower estimate, it is easy to see that in the span of a decade each abacus manual used in a school reached over a hundred pupils. As a consequence, abacus manuals used within schools often had a readership of a few generations of abacus masters, and an audience of several hundreds of students. This makes abacus manuals distinctively public sources and, therefore, a particularly suitable source of evidence to study the diffusion of the mathematical skills they contain. Moreover, as abacus schools preceded the apprenticeship stage, they provided a foundational mathematical training to a wide range of would-be merchants and practitioners.

2.3 The diffusion of practical arithmetic in Europe

Practical arithmetic manuals were mostly used within schools, and their publication in a new city tended to be associated with the foundation of practical arithmetic schools. This is true not only for the Italian tradition of abacus mathematics, but for the entire European tradition (which we will call, more generally, the European tradition of practical arithmetic). Figure 1 shows the intertemporal patterns of diffusion of practical arithmetic manuals in Europe.

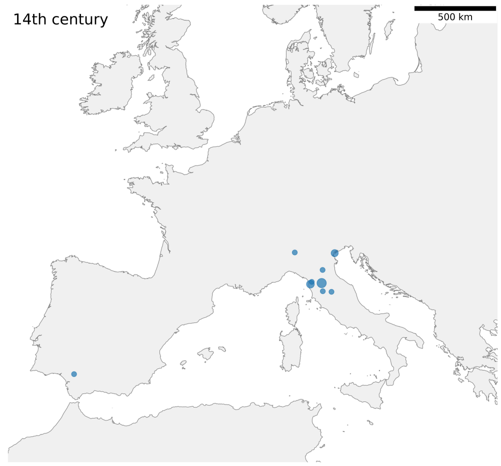
As discussed in detail in Section A.2, practical arithmetic manuals spread incrementally from the Mediterranean to northern Europe between the 14th and the 16th centuries. The chronology of their diffusion shows that data on practical arithmetic manuals does not provide a measure for economic activity in general, but rather a proxy for the spread of specific commercial skills. If this were not the case, we would observe practical arithmetic manuals with Indo-Arabic numerals appear simultaneously in the main European trading and industrial centers of the 13th and 14th centuries—i.e. not only in Italy and southern Europe, but

9. While Grendler estimated an average enrollment of 25 to 40 pupils per abacus school, Van Egmond argued that this number ranged between 40 and 50 students per school.

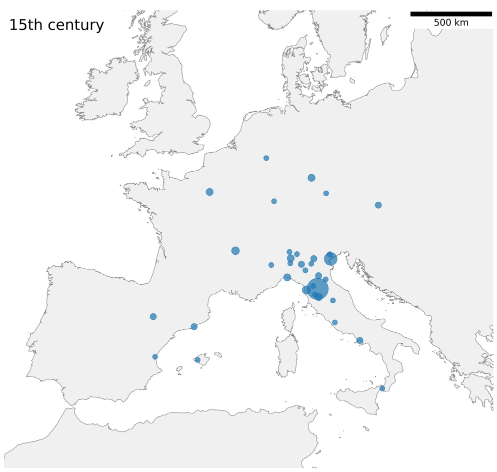
Figure 1: Diffusion of Indo-Arabic manuals across European cities.

(a) 13^{th} century (1200–1299)

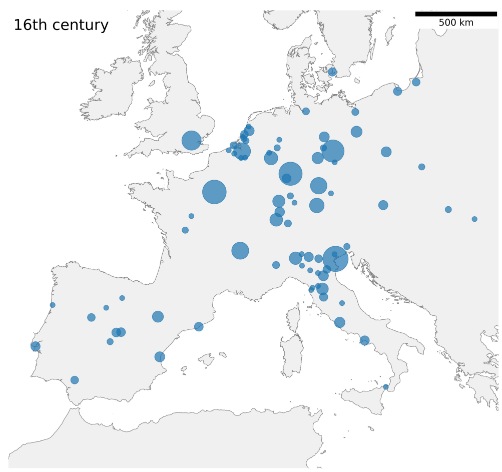
(b) 14^{th} century (1300–1399)



(c) 15^{th} century (1400–1499)



(d) 16^{th} century (1500–1599)



Notes: The circle size is proportional to the number of manuals published in the city in that century.

also in the Low Countries, Hansa cities, and England. Practical arithmetic manuals, instead, appeared in these areas only in the 16th century. As the exchange of ideas and the movement of people were the key channels through which this practical knowledge spread, the European diffusion of practical arithmetic did not follow the maritime routes of international trade, but rather inland channels based on proximity (Danna 2026), as can be observed in figure 1.

This may appear as a rather long diffusion pattern, but we need to note that many general-purpose technologies exhibited slow diffusion patterns and long lags before their economic potential was realized (David 1990; Crafts 2004; Comin and Hobijn 2004; Dittmar 2011). These lags also characterized our historical setting.[10] Besides, while Indo-Arabic numerals were essential for advanced calculation, alternative numerical tools (such as the counter abacus) were adequate for simpler operations, including addition and subtraction with natural numbers. As a result, early modern Europeans employed a plurality of numerical tools, selecting those best suited to the task at hand (Otis 2024). Moreover, there is a broad consensus in the historiographical literature that the commercial revolution techniques took centuries to spread beyond the Alps. The virtual monopoly over international merchant-banking held by Italians for a remarkably long time arguably contributed to a slower pace of diffusion (Spufford 1988). Not only the geography of international banking reflected the dominant position of Italian cities in this sector, but the business of marine insurance and bills of exchange across European cities remained dominated by Italian merchants until the 16th century (Kellenbenz 1986; Tognetti 2015; Denzel 2025).

Italian merchant-bankers were trained in Italy, where they attended vocational schools in which they were taught practical arithmetic with Indo-Arabic numerals, and *only later* traveled abroad to do business, with strong incentives to maintain control of highly profitable techniques.[11] The first non-Italian actors to break this dominant position were merchants from southern Germany who moved to Italy (mainly to Venice) in the second half of the 15th century not as established traders seeking commercial opportunities, but as young

10. Eyeglasses, for example, followed a remarkably similar spread to Indo-Arabic numerals. After their first production in Italy in the late 13th century, they were quickly traded across the European continent (Ilardi 2007). However, the manufacturing of spectacle-quality glass was a highly protected industry that took much longer to spread beyond the Alps, reaching northern Europe only in the 16th century. Only two spectacle-makers are documented as having been active in London in the 15th century; both were Dutch and assembled lenses produced abroad (Ilardi 2007, 133-134). The manufacturing of Venetian-style transparent glass spread to the Low Countries only in the 16th century, as it was introduced in Antwerp in 1537 and in Amsterdam in 1571 (Ilardi 2007, 141-144).

11. Note that showing that bills of exchange were issued in London in the 15th century does not imply that English merchants adopted such an instrument; rather, it indicates only that Italian companies had branches in London, where their agents (who had moved there from Italy after their training) issued such instruments (Bolton and Guidi-Bruscoli 2021).

apprentices seeking professional training there. Jacob Fugger, Lucas Rem and Matthäus Schwarz are a few examples of a number of German merchants who moved to Italy in their teens to master advanced financial and accounting practices during this period (Braunstein 1992; Häberlein 2012). This first generation of German international merchant-bankers then backed the foundation of practical arithmetic schools in their original cities. The first *Rechenschulen* served as the institutional arrangement to secure the diffusion across Upper German mercantile communities of the cutting-edge business education first acquired abroad by these early adopters (Braunstein 2016, 342–44; Weissen 2000).

The transmission of practical arithmetic across cities required, first, that the local mercantile community was exposed to advanced commercial practices used abroad and, secondly, that cities decided to invest in the intergenerational transmission of these skills through the foundation of vocational schools. The key channel through which this knowledge spread was often the migration of skilled people. As a consequence, it is common to find that the first practical arithmetic masters active in a city moved (or were called) to that city from an area of previous adoption with the aim of spreading mathematical knowledge.[12] In some cases, the very first adopters of advanced commercial techniques were themselves the authors of the first manuals in a new area. After learning these techniques abroad, these early adopters wrote texts to foster their dissemination in their area of origin through local practical arithmetic schools.[13]

The appearance of a practical arithmetic manual, therefore, started when merchants decided

12. The first master documented in the north of Italy—Maestro Lotto, who was appointed as public abacus master in Verona in 1284—moved to that area from Tuscany, as he was originally from Florence (Black 2007, 227). German *Rechenmeister* played a key role in the diffusion of the teaching of practical arithmetic in the Low Countries. For example, lists of practical arithmetic masters in 16th-century Antwerp report German names in around 50% of cases (Meskens 2013, 91–94). The diffusion of practical arithmetic to the northern Low Countries, and especially to Amsterdam, was triggered by the migration of Antwerp's Protestant citizens—that included a number of *rekenmeesters*—in the aftermath of the Dutch revolt (52–55). Humphrey Baker's *The welspring of sciences* (1574) attests that, in the third quarter of the 16th century, practical arithmetic skills were still mainly taught by masters coming from the continent (Otis 2017).

13. Fibonacci's preface to his *Liber abaci* can be read as a deliberate choice to stimulate the introduction of Arabic arithmetic in the western-Latin world (Giusti and D'Alessandro 2020, 4). Jan Ympyn Christoffels, who wrote the first treatise on bookkeeping published in northern Europe, had spent a decade in Venice, where he had been sent by his merchant father to learn business practices and accounting (Yamey and Bywater 1982).

to invest in the intergenerational transmission of advanced mathematical techniques. Working in a way comparable to a general-purpose technology, Indo-Arabic numerals enabled the diffusion of advanced commercial techniques and had a significant (lagged) effect on economic growth.

3 Data and variables

3.1 Practical arithmetic manuals

We rely on a new dataset of over 1290 practical arithmetic manuals that was compiled relying on previous literature and extensive archival research. The dataset gathers over 350 manuscripts and over 940 printed manuals written from the late 13th century to 1600. These manuals were written by more than 340 authors, who were active in over 120 cities of western Europe, scattered between Austria, Denmark, England, France, Germany, Hungary, Italy, the Low Countries, Poland, Portugal, Spain, and Switzerland. The database records texts written in the following languages: Castilian, Catalan, Danish, English, French, German, Greek, Hebrew, Hungarian, Italian, Latin, Netherlandish, Polish, Portuguese, and Provençal. This database provides the first reconstruction of the European tradition of practical arithmetic and shows that the diffusion of these texts was a continuous and piecemeal process. The evidence provided in this database contributes to existing literature in two ways. First, as it covers a period of four centuries, it provides a long-run analysis which is based not on estimates of book production, but on clearly identifiable primary sources (Baten and Van Zanden 2008). Second, as it is based on both manuscript and printed sources, this database makes it possible to identify a continuous transmission of mathematical knowledge from the commercial revolution of the 13th century to the onset of the European 'little divergence'—i.e., the divergence of north-western Europe (the Low Countries and England) relative to southern and central Europe, starting from the 16th century (De Pleijt and Van Zanden 2024).[14]

14. Moreover, for the reasons discussed above, this database covers a particularly significant subset of total European book production, providing a good proxy to study the spread of commercially-oriented knowledge. This tradition of vernacular practical arithmetic, in fact, documents the long-run roots of the mathematical

While manuscript sources are considered individually (with each manuscript being considered as an independent document), printed manuals have been recorded at the book-edition level. This choice is due to three independent reasons: (1) the different characteristics of manuscript and printed sources, (2) the varying social circulation of European practical arithmetic manuals, both manuscript and printed, and (3) the nature of the evidence extant, as discussed in appendix A.1.

Books offer a reliable source of evidence to reconstruct the diffusion of knowledge. The cost of shipping printed books was often sufficiently high to make it more convenient to reprint a text in a new city, rather than to ship large numbers of volumes across cities, thus printed books spread through new editions rather than through inter-city trade (Dittmar 2011). These considerations also apply to manuscript texts, with the addition that the high flexibility of the manuscript medium made these sources less standardized than printed documents, reflecting the interests and features of the local context in which they were written, and making them even less fit for inter-city trade than printed books.

3.2 Variables and samples

We exploit this novel dataset to investigate the importance of the diffusion of practical mathematics for the growth of European cities. Due to the lack of per capita income data for historic cities, we follow Dittmar (2011) and use population data as a proxy for economic growth. Since the pioneering work by Bairoch et al. (1988), the population of European cities in pre-modern Europe is, indeed, well documented at the century-level. To perform our analysis, we combine the Buringh (2021) dataset on the population of European cities from the 8th to the 21st century[15] with more detailed population estimates for Italian cities by Malanima (1998) and Malanima (2011).[16] Our estimation results are, however, robust to the use of different data samples, as we discuss in section 6.

knowledge that Kelly and Ó Gráda (2022) have recently discussed as a crucial factor in the industrial revolution.

15. These data contain a large share (almost 50%) of observations obtained by simple linear imputation. Our choice was to retain only the more reliable data, and hence we removed imputed observations.

16. Dataset available at: Italian urban population 1300–1861, Paolo Malanima, 2005.

To capture the long-run effects of practical arithmetic manuals on urban growth, our analysis focuses on population outcomes in the centuries following publication. Given that our manual data covers the 13th–16th centuries, we examine population levels in the 14th–17th centuries, using a standard time lag in the OLS regressions. Moreover, we consider only the twelve countries (following the 20th-century classification) that are present in our manuals dataset: Austria, Denmark, England, France, Germany, Hungary, Italy, the Low Countries, Poland, Portugal, Spain, and Switzerland. The resulting dataset provides information about population growth in 827 European cities, with 1799 city-century total observations. We select the manuals published in these 827 cities from the entire database of practical arithmetic manuals, and we obtain 1040 texts written in 94 cities. Table 1 provides more details about the distribution of European cities in our sample across countries and centuries. The number of observed cities grows considerably over the centuries: while we have information on population growth for only 282 cities in the 14th century, the same data are available for 732 cities in the 17th century.

To better estimate the effect of the diffusion of practical mathematics on population growth, we also consider in the analysis a number of aspects that could affect urban economies in pre-modern Europe. First of all, we control for the adoption of printing technology in the city. The first European movable-type printing press was introduced by Johannes Gutenberg in Mainz around 1450. In the following years, printing technology spread across the main European cities and substantially contributed to the decrease in book prices and to the dissemination of merchants' manuals and intellectual works. Dittmar (2011) finds a positive impact of the adoption of printing in 1450–1500 on city growth during the 16th century. In our analysis, we also consider the presence of printing in European cities during the 16th century, so as to cover our entire period of interest. To create an indicator of the presence of at least one printing press in a city, we rely on the *Incunabula Short Title Catalogue*, for what concerns the books published up to 1500, and on the *Universal Short Title Catalogue*,

Table 1: Historic cities and historic cities with practical arithmetic manuals across centuries and 20th-century countries.

Century	Total historical cities				Cities with manuals			
Country	14th	15th	16th	17th	13th	14th	15th	16th
Austria	1	1	8	10	0	0	1	1
Denmark	1	1	2	7	0	0	0	1
England	15	19	34	54	0	0	0	1
France	32	30	45	61	0	0	2	4
Germany	36	46	84	102	0	0	4	22
Hungary	1	4	4	4	0	0	0	2
Italy	141	126	159	246	2	8	23	22
Low Countries	25	36	49	53	0	0	0	12
Poland	3	4	18	21	0	0	0	3
Portugal	2	2	7	6	0	0	0	2
Spain	20	23	71	161	0	1	4	10
Switzerland	5	6	6	7	0	0	0	2
Total	282	298	487	732	2	9	34	82

Notes: The left-side table reports the number of historic cities in which we observe population estimates at the beginning and the end of the century across time and 20th-century countries. The bottom line summarizes the total number of cities per century. The right-side table reports the size of the subset of cities with at least one practical arithmetic manual across centuries and 20th-century countries.

for data after 1500.[17]

In addition to controlling for the presence of printing technology, we follow previous literature (Acemoglu et al. 2005; Dittmar 2011) and include in the analysis a series of dummy variables that capture cities' cultural, political, and geographical status. Firstly, we include a variable indicating the presence of a university in the city, as a proxy for its intellectual activities and cultural development. This variable may vary across centuries. We extract this information from De Ridder-Symoens and Rüegg (1992), and obtain observations on 105 different universities in our sample of European cities during the 14th–17th centuries. To capture the political status of cities, we define a dummy variable indicating whether

17. The *Incunabula Short Title Catalogue* is available at: ISCT, maintained by the British Library (2016). In February 2022, the database collected 26297 printed books published before 1500 in 283 European cities. The *Universal Short Title Catalogue*, instead, is available at: USTC, hosted by the University of St Andrews (2022). In February 2022, we identified 311665 printed books published between 1500 and 1600 in 448 cities.

the city was a state capital. We integrate the data on state capitals in pre-modern Europe by Bosker et al. (2013) with historical sources to check whether cities were state capitals throughout the century and not only at its beginning. As expected, this variable changes over time, and we count 54 capitals in our period of observation. Concerning geographical location, we control for features—such as the presence of roads and ports—that may favor trade with other cities. We obtain information about the presence of Roman roads in the proximity of each city by integrating data by Bosker et al. (2013) with the geolocalization of Roman roads.[18] We also define a variable accounting for the presence of a navigable river close to the city, once again relying on the information collected in Bosker et al. (2013). This data has been compared with the information about European cities in Buringh (2021) and checked manually for the remaining cities. Finally, we identify cities with seaports by distinguishing between Mediterranean, Atlantic, Baltic, and North Sea ports. We use data by Acemoglu et al. (2005) and Buringh (2021) as main sources of information on these city characteristics. Table A1 in appendix A.3 summarizes descriptive statistics concerning dependent, independent, and control variables in our panel dataset. Each observation refers to city-century pairs.

4 Practical arithmetic and city growth

To estimate the importance of the diffusion of practical mathematics in pre-modern European economies, we initially consider city population as a proxy for economic development and its association with the diffusion of practical arithmetic manuals across European cities during the previous century (13th–16th centuries). We, therefore, estimate the following model by an Ordinary Least Squares (OLS) regression, pooling together all centuries of interest:

$$\log(\text{End-of-century pop}_{c;\,t}) = \beta_0 + \beta_1 \,\text{Manuals}_{c;\,t-1} + \gamma \mathbf{X}_{c;\,t} + \delta_t + \phi_a + \epsilon_{c;\,t}, \tag{1}$$

18. We measure the distance between cities and Roman roads and consider cities no more than 2.5 km away from those roads. The geolocalized data are from Harvard Center for Geographic Analysis - Roman World.

where c is a European city, t is the century (from the 14th to the 17th century), a is the country the city belongs to, $\mathbf{X}_{c;t}$ is a vector collecting the set of controls at the city level, δ_t and ϕ_a capture, respectively, century and country (following the 20th-century classification) fixed effects, and $\epsilon_{c;t}$ is the error term. In all estimates, we report clustered standard errors by country. To better understand the economic importance of the diffusion of practical arithmetic manuals in a given city, we provide a set of alternative variables to measure the presence of manuals in that city: the *number of manuals* in the previous century, the *number of manuals in the second half* of the previous century, and, finally, a dummy variable that captures *the presence of at least one manual* in the previous century. Because the distributions of the number of manuals are skewed, these variables are log-transformed and included with a lag to mitigate reverse causality concerns in this econometric setup.

Following previous studies, controls at the city level include the presence, during century t, of a university or state capital, as well as access to navigable rivers or seaports. They also include a dummy variable that signals the presence of the technology of printing during the century in which we observe practical arithmetic manuals ($t-1$). Finally, we control for the logarithm of population at the beginning of the century to account for the initial level of economic development.

Table 2 reports the result in specifications (1)–(3). For all the indicators capturing practical arithmetic manuals, we observe a positive and significant association between the diffusion of texts and the city population at the end of the following century. More in detail, a 10% increase in the number of manuals in the century $t-1$ leads to a 1.24% increment in city population during century t. This association is stronger when considering texts from the second half of the century: a 10% increase in the number of manuals in the second half of the century is associated with a 1.70% increase in city population. The presence of at least one manual in the city during the previous century increased the city population by 15.60%. To address possible sources of endogeneity, such as the omitted variable bias, and because poolability tests for fixed effects suggest the presence of city effects, we exploit the panel nature of our database, and we control for unobserved urban characteristics by replicating the previous analysis with city and century fixed effects:

$$\log(\text{End-of-century pop}_{c;\,t}) = \beta_0 + \beta_1\,\text{Manuals}_{c;\,t-1} + \gamma\mathbf{X}_{c;\,t} + \delta_t + \phi_c + \epsilon_{c;\,t}, \tag{2}$$

where c is a city, t is the century, $\mathbf{X}_{c;\,t}$ is a vector collecting the set of controls at the city-century level, δ_t and ϕ_c capture, respectively, time and city fixed effect, and $\epsilon_{c;\,t}$ is the error term. All estimations report clustered standard errors at the city level. Overall, panel estimates confirm previous results. Even when we control for unobserved urban characteristics, we find a positive and significant association between practical arithmetic manuals and city growth, as reported in table 2 in specifications (4)–(6). Indeed, an increase of 10% in the number of texts in circulation in a given century is associated with an increment of 0.82% in city population in the following century. As before, the manuals published in the second half of the century are particularly relevant: an increment of 10% in the number of manuals in circulation in the second part of the previous century leads to a 1.12% increase in urban population. However, in this case, the association estimated through the dummy variable is positive but not significant.

4.1 Event study analysis

To quantify more precisely the long-run effect of practical arithmetic manuals' adoption on city growth, we design a quasi-experiment based on a difference-in-differences (DiD) approach. In our case, treatment, i.e. instances of the first appearance of a practical arithmetic manual in a city, is distributed over four centuries (13th–16th). As there are more than two time periods, we face a setting with staggered treatment rollout. Recent advancements in the DiD literature highlight several issues associated with a simple generalization of the DiD methodology to staggered treatment timing and suggest new methodologies to obtain consistent estimates (see, among others, Baker et al. 2022 and Roth et al. 2023).

To correct for the bias induced by the staggered nature of practical arithmetic manual adoption that would arise employing a standard DiD, we use the approach developed by Callaway

Table 2: Practical arithmetic manuals and the city population at the end of the following century.

Dependent Variable:	log(End-of-century population$_t$)					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Nb. of manuals$_{t-1}$+1)	0.124***			0.082**		
	(0.028)			(0.035)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.170***			0.112***	
		(0.030)			(0.041)	
Manuals dummy$_{t-1}$			0.145***			0.085
			(0.052)			(0.055)
Printing press$_{t-1}$	0.122***	0.119***	0.125***	0.036	0.032	0.043
	(0.033)	(0.033)	(0.034)	(0.043)	(0.043)	(0.044)
University	0.084**	0.087**	0.087**	0.004	0.009	0.004
	(0.040)	(0.040)	(0.041)	(0.064)	(0.064)	(0.064)
Capital	0.413***	0.409***	0.423***	0.278***	0.274***	0.276***
	(0.067)	(0.066)	(0.069)	(0.099)	(0.098)	(0.100)
log(Beginning-of-century population$_t$)	0.693***	0.691***	0.701***	0.294***	0.291***	0.299***
	(0.019)	(0.019)	(0.019)	(0.038)	(0.037)	(0.038)
Geographical controls	Yes	Yes	Yes	No	No	No
Country FE	Yes	Yes	Yes	No	No	No
City FE	No	No	No	Yes	Yes	Yes
Century FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,799	1,799	1,799	1,799	1,799	1,799
R^2	0.735	0.736	0.733	0.895	0.895	0.895
Within R^2	0.710	0.711	0.708	0.113	0.116	0.109

Notes: This table presents pooled OLS estimates of equation 1 (specifications 1–3) and panel fixed-effect OLS estimates of equation 2 (specifications 4–6). The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, state capital, and geographical controls (Roman roads, navigable rivers, and seaports). We also control for the logarithm of the city population at the beginning of the century t, and we add country (in pooled OLS), city (in panel fixed-effect OLS), and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

and Sant'Anna (2021) that generalizes the parallel trends assumption to multi-period settings. Callaway and Sant'Anna (CS) is a highly flexible approach that ensures the use of clean controls (i.e., cities that did not receive the treatment before the considered point in time) also in the case of staggered adoption, and allows treatment effect heterogeneity by computing individual cohort-time-specific treatment effects and then aggregating them to produce overall effect estimates. Different from other staggered DiD estimators, CS can be applied to unbalanced panel data and is robust to parallel-trend assumptions that hold conditionally on covariates. Since we are dealing with historical data at the city level and population data time series might contain missing values—especially for early centuries—resulting in an unbalanced panel, this estimator is particularly suited for our exercise. Unlike the panel fixed-effect analysis, this approach allows us to select an adequate control group of cities and obtain a more precise estimate of the effect of practical arithmetic adoption. Since the adoption is staggered, whenever the control group includes already treated cities (as in our previous analysis), we are likely underestimating the impact of practical arithmetic on city growth.

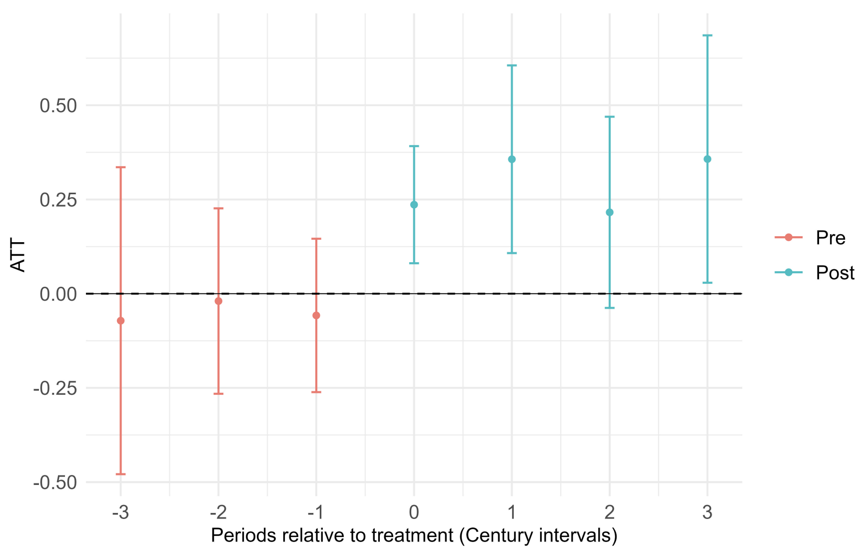
In this analysis, we extend the population time series to cover, when possible, the period from the 12th to the 19th century. We consider as treated only cities that adopted practical arithmetic manuals in the 14th–16th centuries, since only two cities adopted those manuals in the 13th century.[19] The outcome variable, as in the previous exercise, is the logarithm of the city's population at the end of the century to account for the lagged effect of practical arithmetic on city growth. The control group comprises not-yet-treated cities, which are more likely to have a similar pre-trend in population than never-treated cities. We then apply CS in an event-study framework and estimate the following coefficients for each period e ($-3 \leq e \leq 3$) by comparing treated and not-yet-treated cities:

19. To ensure clean controls in the analysis, we remove Pisa and Perugia from our sample. However, results are unchanged if we include those cities as treated in the 13th century. Moreover, we considered only cities with at least one population observation in the two centuries before the treatment to account more precisely for pre-trends and obtain more robust estimates. This results in the exclusion of 5 treated cities (Oppenheim, Pforzheim, Regensburg, Reggio Calabria, Zwickau).

$$\theta_{es}(e) = \sum_{g \in \mathcal{G}} \mathbb{1}\{g + e \leq \mathcal{T}\} P(G = g | g + e \leq \mathcal{T}) ATT(g, g + e), \tag{3}$$

where $\mathcal{G}$ is the set of possible treatment periods, G is the time period when a unit becomes treated, $\mathcal{T}$ is the number of periods ($t \in \{1, \ldots, \mathcal{T}\}$), P is the probability of being treated in a given period, $ATT(g,t)$ is the group-time average treatment effect. It is defined as $ATT(g,t) = \mathbb{E}[Y_t(g) - Y_t(0)|G_g = 1]$, where G_g is a binary variable equal to one if a unit is first treated in period g and Y_t is the outcome in period t.

Figure 2: Event study on the city's population at the end of the century.



Notes: The figure shows DiD coefficients estimated following equation 3 for European cities. Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of city population at the end of the century. Treatment occurs at time 0. P-value for pre-test of parallel trends assumption (Ward test): 0.20. Confidence intervals are 95%. Standard errors are clustered at the city level.

The results, reported in figure 2, show that the first appearance of a practical arithmetic manual has a positive and significant effect on the city population in the following centuries. Results can be interpreted in causal terms under the no-anticipation and parallel trends assumptions. If we relax the no-anticipation assumption by allowing the treatment effect to display one period before the adoption date, the point estimates remain stable and consistent with our baseline results, despite a slight increase in standard errors and a marginal loss of statistical significance in the event study due to the inclusion of more distant population data as baseline (see robustness checks in section 6). Moreover, the results support the validity of the no-pre-trend assumption. First, the pseudo-treatment effects for the pre-treatment periods are small and statistically insignificant. Second, the Wald pre-test fails to reject the null hypothesis of parallel trends (p-value > 0.05), confirming that the treated and control cities were on comparable growth trajectories before adoption. The observed city growth can, therefore, be interpreted as a consequence of the introduction of practical arithmetic manuals rather than pre-existing trends. Specifically, cities that adopted a practical arithmetic manual during the century t were, on average, 26.64% larger by the end of the century compared to similar cities that had not yet adopted manuals. The magnitude of the effect increases in the century following adoption, reaching a 42.87% increase in population, consistent with historiographical evidence of a gradual, multi-generational diffusion and institutionalization of practical arithmetic knowledge. This effect is persistent in the following centuries (42.95% after three centuries). Aggregating across all post-treatment periods, the overall simple Average Treatment Effect on the Treated (ATT) is 0.292 (std. error: 0.090). This corresponds to a statistically significant average increase of 33.91% in the city population.

5 The mechanism

Arithmetic manuals appeared as a result of deliberate urban policies to invest in local practical knowledge through arithmetic schools. This possibly dates back to the very origin of the tradition of abacus mathematics, as in 1241 the Republic of Pisa provided Fibonacci with an annual salary in compensation for his knowledge, identifying him as a 'magister' (Ulivi 2011, 256–57). In the 14th century, the city of Pistoia justified its abacus school foundation stating that without practical arithmetic its merchants and artisans could not have run their businesses in appropriate ways ("Sine scientia abaci mercatores et artifices utiliter et bene se exercere non possunt") (Grendler 1989, 22).[20] We therefore focus on the training of

20. Similar developments are observed in several other European cities, such as Lyon. In the second half of the 15th century, Lyon founded a number of practical arithmetic schools to boost the skills of its merchants vis-à-vis foreign merchant-bankers. The Lyonnaise merchant François Garin, for example, showed

mercantile classes as the primary mechanism driving city growth.

5.1 The training in practical arithmetic of notable merchants

To explore the mechanisms of human capital formation, we complement the previous findings with an analysis of notable merchant birth cohorts, drawing on the Notable People dataset developed by Laouenan et al. (2022).[21] As these records collect only medieval merchants who are known in the present time, they represent an underestimation of the actual number of merchants in European cities. However, they can be interpreted as a good proxy for flourishing commercial activities in observed cities. Since pupils typically completed practical arithmetic schools when they were around 15 years old, we construct the birth cohorts by adding a 15-year lag to the merchants' birth years to identify the population of students finishing vocational training at the time of treatment. In doing so, we observe a population of 15-year-olds who would grow up to become prominent business leaders. The granularity of these data also allows us to investigate the role of practical arithmetic on a shorter time frame. Since both the appearance of practical arithmetic manuals and the birth of notable merchants are rare events, we aggregate data into discrete 20-year intervals to ensure more consistent estimates. For similar reasons, we limit the sample to 20-year intervals after 1300. Given that practical arithmetic manuals were necessary tools for the vocational training of merchants, we expect that their impact on the mercantile class will appear within a shorter temporal lag than their influence on aggregate urban growth estimated in the event study of Section 4.1.

We first estimate panel fixed-effect models by OLS regressions, investigating the association between the publication of practical arithmetic manuals and the number of 15-year-old would-be notable merchants over the following 20 years. As the dependent variable is ex-

the awareness of this gap in competencies among the local mercantile community in the 1460s, when he urged his son to master the new mathematical techniques (Dubuis 1978, 94–95).

21. The database collects cross-verified records on 2.29 million individuals (3500BC-2018AD), providing information on their date and location of birth and death, as well as occupation. We identify as merchants all European notable people with main occupation (level 2) classified as 'Corporate/Executive/Business (large)'. Records are checked manually and include famous merchants such as Francesco Datini, Giovanni di Bicci de' Medici, Anton Fugger, and Thomas Gresham, who are all known to have had a training in practical arithmetic (Danna 2026).

tremely skewed, with only 1.2% of records across 94 cities that are different from zero, we consider the logarithm transformation of the variable plus 1. To verify that this scaling does not influence our baseline conclusions, we also estimate a Linear Probability Model (LPM) using a dummy variable for the presence of 15-year-old notable merchants.

$$\text{Merchants}_{c;\,t+1} = \beta_0 + \beta_1\,\text{Manuals}_{c;\,t} + \gamma \mathbf{X}_{c;\,t} + \delta_t + \phi_c + \epsilon_{c;\,t}, \tag{4}$$

where c is a city, t is the 20-year interval, $\text{Merchants}_{c;\,t+1}$ indicates, alternatively, the log-transformed number of 15-year-old notable merchants at time t+1 and and the corresponding dummy variable, $\text{Manuals}_{c;\,t}$ indicates the main independent variables (i.e., the log-transformed number of manuals and the manual dummy) at time t, $\mathbf{X}_{c;\,t}$ is a vector collecting the set of controls at the city-20-year-interval level, δ_t and ϕ_c capture, respectively, time and city fixed effect, and $\epsilon_{c;\,t}$ is the error term. All estimations report clustered standard errors at the city level. The set of controls includes the presence of printing technology, the presence of a university or state capital, the log-transformed number of 15-year-old merchants in the 20-year interval, and the log-transformed number of 15-year-old notable people in the 20-year interval. By controlling for the latter variable, we ensure that our analysis captures the relative importance of mercantile professions rather than a general increase in city prestige or an expansion of historical records. Alternative specifications control for the growth of notable people between the two periods and the estimated population growth (based on the century population growth—see robustness checks in section 6).

As shown in table 3, our findings highlight a positive and significant association between the presence of practical arithmetic manuals and the number of 15-year-old notable merchants in the following 20 years, across all specifications. Specifically, the adoption of practical arithmetic manuals results in a 6.72% increase in the number of notable merchants in the next 20 years (a 10% increase in the number of those manuals results in a 0.64% increase). Moving to the LPM, we observe that adopting at least one manual increases the probability of merchant presence over the next 20 years by 7.0 percentage points and that moving from zero to one manual increases it by 4.51 percentage points.

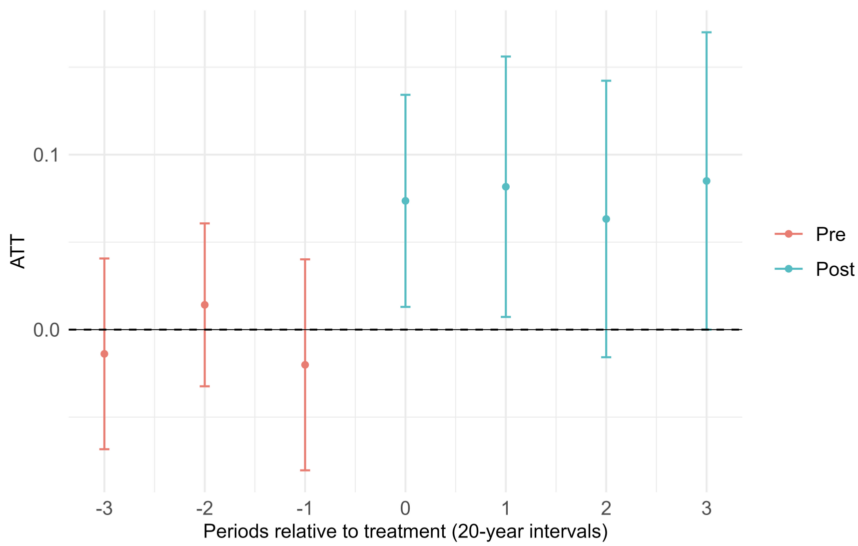
Table 3: Practical arithmetic manuals and the number of 15-year-old notable merchants in the following 20 years.

Dependent Variables:	log(Nb. of merchants$_{t+1}$+1)		Merchants dummy$_{t+1}$	
	(1)	(2)	(3)	(4)
log(Nb. of manuals$_t$+1)	0.064***		0.065***	
	(0.018)		(0.020)	
Manuals dummy$_t$		0.065***		0.070***
		(0.021)		(0.024)
Printing press$_t$	0.002	0.003	0.006	0.007
	(0.004)	(0.004)	(0.005)	(0.005)
University	-0.015*	-0.015	-0.025***	-0.025***
	(0.009)	(0.010)	(0.009)	(0.009)
Capital	-0.006	-0.007	-0.006	-0.007
	(0.009)	(0.009)	(0.012)	(0.013)
log(Nb. of merchants$_t$+1)	0.112**	0.125**	0.043	0.055
	(0.056)	(0.060)	(0.046)	(0.050)
log(Nb. of notable people$_t$+1)	0.016***	0.017***	0.017***	0.018***
	(0.004)	(0.005)	(0.004)	(0.005)
City FE	Yes	Yes	Yes	Yes
20y interval FE	Yes	Yes	Yes	Yes
Observations	11,578	11,578	11,578	11,578
R^2	0.350	0.344	0.282	0.279
Within R^2	0.050	0.042	0.027	0.022

Notes: This table presents panel fixed-effect OLS estimates of equation 4. The dependent variables are the logarithm of the number of 15-year-old notable merchants + 1 at time $t+1$ and a dummy variable equal to one if there is at least one 15-year-old notable merchant at time $t+1$. t indicates the 20-year intervals in the 14th–16th centuries. The $Manuals$ variables identify practical arithmetic manuals at $t-1$. Control variables include the presence of printing presses, a university, and a state capital. We also control for the number of 15-year-old notable merchants and notable people at time t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

To test the plausibly causal, medium-run effect of practical arithmetic manuals on the number of notable merchants, we reproduce the CS event study analysis proposed in section 4 applied to the 20-year-interval balanced panel. We consider the log-transformed number of 15-year-old notable merchants over 20 years as the outcome variable. The control group consists of the cities that have not yet been treated. To properly compare cities, we control for the initial (log-transformed) number of notable 15-year-olds in the city as a proxy for both the relative importance of the city and the availability of historical records.

Figure 3: Event study on the city's 15-year-old notable merchants in 20 years.



Notes: The figure shows DiD coefficients estimated following equation 3 for European cities. Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of the number of 15-year-old notable merchants plus 1 in the 20-year interval. Treatment occurs at time 0. Pre-treatment is conditional on the number of notable 15-year-olds in the city. P-value for pre-test of parallel trends assumption (Ward test): 0.39. Confidence intervals are 95%. Standard errors are clustered at the city level.

Figure 3 reports the result. Importantly, we do not observe any pre-trends. The non-significant pseudo-treatment effects for the pre-treatment periods show that, before the first appearance of a practical arithmetic manual, cities in the treated and control groups display a similar trend in the number of 15-year-old notable merchants. The absence of a pre-trend, once we control for the initial number of notable people, is confirmed by the failure of the Wald pre-test in rejecting the null hypothesis of parallel trends. Moreover, point estimates are consistent with our main results even if we relax the no-anticipation assumption by allowing the treatment effect to display one period before the adoption interval (see robustness checks in section 6). The main results show that the first publication of a practical arithmetic manual positively increases the number of people who will become notable merchants in the next few years. This finding is in line with the historiographic evidence suggesting that practical arithmetic manuals were adopted by practical arithmetic schools providing vocational training for future practitioners. The adoption of these texts increases the number of 15-year-old would-be notable merchants by 7.64% over 20 years, compared to cities that have not yet been treated. The effect is persistent in subsequent periods, with an expansion of 8.51% and 8.87% over 40 and 80 years, respectively. The overall ATT is 0.076 (std. error: 0.024), corresponding to a statistically significant average increase of 7.90%.

5.2 Arithmetic manuals and printed commercial literature

The same mechanism can also be tested by examining the association between the introduction of practical arithmetic manuals and later texts specifically designed to provide advanced business skills to merchants, e.g., accounting, tables of conversion, information on customs and duties, etc. A list of printed books for merchants, including the city and year of edition, is collected in the catalog *Ars Mercatoria: Handbücher und Traktate für den Gebrauch des Kaufmanns* (Hoock et al. 1991). Among the 827 cities considered in our main dataset, 153 display at least one text listed in *Ars Mercatoria*. Because (1) practical arithmetic was the fundamental background knowledge of mercantile textbooks, and hence (2) training in advanced commercial skills typically followed that in arithmetic, we expect that practical arithmetic manuals drive the emergence of *Ars Mercatoria* texts.

To test this hypothesis, we first replicate the panel fixed-effect analysis performed on the data of notable merchants by grouping the data into 20-year intervals due to the sparsity of records. Our dependent variables are, alternatively, a logarithm transformation of the *Ars Mercatoria* texts in the next 20 years plus 1, and a dummy variable equal to one when we observe at least one *Ars Mercatoria* text in the following 20 years.[22] Regression models are estimated through OLS and LPM, respectively. As we consider only printed books, we limit the analysis to the period after the emergence of the printing press (around 1450). In addition to the control variables included in previous regressions, as the *Ars Mercatoria* texts are all printed books, we control for the log-transformed number of printed books. Alternative specifications control for the growth of notable people between the two periods

22. The distribution of texts listed in *Ars Mercatoria* across cities is very skewed, with only 5.5% of city-20-year-interval records different from zero.

as a proxy for changes in city prestige and the estimated population growth (based on the century population growth–see section 6). More details are in appendix B.1.

The results (see table B1) show a strong positive and significant association between the presence of practical arithmetic manuals and the number of *Ars Mercatoria* texts published in the following 20 years. An increase of 10% in the number of manuals is associated with a growth of 2.05% in the *Ars Mercatoria* texts. In other words, the presence of at least one manual is related to a 13.77% increase in mercantile texts. On the extensive margin, the adoption of at least one manual increases the probability of printing texts listed in *Ars Mercatoria* over the next 20 years by 8.8 percentage points, and moving from zero to one practical arithmetic manual increases this probability by 4.23 percentage points.

Finally, we reproduce the CS event study proposed in the previous sections. Cities treated before the observation period are removed from the analysis. This exclusion allows us to test whether practical arithmetic manuals adopted by cities after the diffusion of the printing press drive the introduction of specialized mercantile printed texts, rather than a flourishing mercantile community influencing the adoption of practical arithmetic manuals. As for notable merchant data, we consider a balanced panel where the outcome variable is the logarithm of the number of *Ars Mercatoria* texts plus 1 in the city in an interval of 20 years, and a control group of not-yet-treated cities. We control for the initial number of printed books to account for the presence of a printing press in the city. The event study (see figure B1) displays non-significant pseudo pre-treatment effects, confirming the parallel trend assumption when paired with the p-value resulting from the Ward pre-test. Overall findings show that the adoption of practical arithmetic manuals results in 51.13% more specialized mercantile texts (overall ATT: 0.413; std. error: 0.077). The effect is persistent over 80 years: it is stronger in the period of adoption (66.80%), but it remains pronounced after 20 (58.09%), 40 (43.46%), and 60 years (37.95%). These results, combined with previous findings on the number of notable merchants and city growth, show that the adoption of practical arithmetic manuals in a city drives the establishment of a successful community of merchants applying advanced commercial techniques, which, in turn, supports city growth.

6 Robustness checks

We report a series of additional tests to stress the robustness of our results on the relationship between practical arithmetic manuals and city growth, as well as additional details and robustness checks on the mechanism.

6.1 Instrumental variable approach

First, we propose a quasi-experimental design based on an instrumental variable (IV) that mirrors the diffusion of practical arithmetic manuals across Europe and across centuries.[23] We generate historical linguistic maps by associating languages with historic states available from the Euratlas project[24] and identify the first practical arithmetic manual in each linguistic area, written in the local vernacular (see table B2). Since practical arithmetic manuals tend to spread by proximity rather than other channels (including maritime trade routes), cities close to diffusion centers are more likely to adopt them. Building on these premises, we employ an instrumental variable based on distance from the centers of first adoption. Different from the IV used by Dittmar (2011) to estimate the economic effect of the printing press (i.e. the fixed distance between adopting cities and Mainz), our IV varies over time. More details on the IV are in Appendix B.2.

The IV, mirroring the diffusion of practical arithmetic manuals, is likely correlated with the number of those manuals in different cities and centuries, as confirmed by first-stage regressions (see table B3). At the same time, the distance from the first center of diffusion in each linguistic area and its variations over centuries are exogenous to the determinants

23. As discussed in previous sections, we observe an incremental diffusion of practical arithmetic manuals from the south to the north of Europe. While central Italy was the center of diffusion for these manuals in the 13th century, new centers of diffusion emerged in the following centuries, corresponding to the first translations of practical arithmetic manuals in regional vernaculars and boosting the circulation of this mathematical knowledge in the new linguistic area. For example, the anonymous *Die maniere om te leeren cyffren*, printed in Bruxelles in 1508, was both the first practical arithmetic manual published in the Low Countries and the first text of this tradition to be written in Dutch. Similarly, the English tradition of practical arithmetic started off with the first translations of French and Netherlandish texts into English (Williams 2012).

24. euratlas.com.

of urban growth. Second-stage estimations support a plausibly causal interpretation of the positive and significant effects of the spread of practical arithmetic on the population of European cities (see table B4). Compared with the panel fixed-effect OLS estimates, these results show a stronger effect of practical arithmetic on growth. An explanation for this difference lies in the instrument's ability to detect fine-grained variations in the adoption and spread of Indo-Arabic numerals in commercial practices. First, the distance from the first center of adoption reduces the attenuation bias resulting from the possible presence of missing practical arithmetic manuals in nearby cities. Second, the number of practical arithmetic manuals published in the city is arguably an underestimate of the actual degree of integration of Indo-Arabic numerals into local commercial practice. By capturing, instead, patterns of diffusion of practical arithmetic across European cities based on distance, the instrument allows us to distinguish between cities with the same number of manuals but with differences in local knowledge assimilation. Results are robust to estimations of falsification tests (see table B5), where we replace the distance from the linguistic centers of diffusion with the distance from the main economic centers of each area.

6.2 Controlling for spatial correlation and spillover effects

We account for potential spatial dependency by introducing Conley standard errors (Conley 1999, 2008) in the panel fixed-effect analysis of the relationship between practical arithmetic manuals and city growth (see table B6). To further evaluate spatial dependence, we estimate Moran's I statistics (Moran 1950) for city-level outcomes and model residuals across our cross-sectional specifications. For all centuries (14th–17th), the observed Moran's I values are consistently negligible, ranging from -0.003 to -0.001. The negligible magnitude of these coefficients confirms the absence of substantive regional clustering or spatial spillovers in city growth. We also investigate potential spillover effects of practical arithmetic manuals by interacting the manuals' count in a city with the log-transformed number of practical arithmetic manuals in neighboring cities at various radii. Results, reported in table B7, show the absence of significant interactions at distances between 20 km and 80 km, and the presence of negative and significant interactions in the 100 km radius, indicating a regional competition effect in the use of arithmetic techniques. This aligns with historiographical evidence reporting that trade between cities at short distances typically involved low-value goods, whereas high-value goods were traded at much greater distances, implying that cities within a 100 km radius from one another would likely be competitors in high-value international trade routes (Cipolla 1956, 56).

6.3 Anticipation and placebo-treatment tests in the event studies

Controlling for adopters' anticipation. The results of the event study can be interpreted as causal under no-anticipation and parallel-trend assumptions. We have already discussed the absence of pre-trends, but we can also relax the no-anticipation assumption to evaluate the robustness of our finding to potential anticipation of treatment (which is unlikely for century-level estimates). In other words, we allow the city to decide to adopt practical arithmetical manuals in advance. The results are qualitatively unchanged (see table B8).

Placebo treatment. We propose two placebo-treatment tests for the event studies. First, we randomize the treatment timing across treated cities. Second, we randomize the treatment assignment, allocating practical arithmetic manuals to non-treated cities. The results show no significant effect of placebo treatment (see table B9).

6.4 Alternative data, sample selections, variables, and controls

Real wages of skilled workers in selected cities. As an alternative indicator of growth, we leverage data on real wages of skilled workers at the decade level (therefore at a more granular level of temporal aggregation) from Serafinelli and Tabellini (2022), based on evidence gathered by Allen (2001). Skilled-worker wages' data are a more direct measure of economic development because practitioners trained in practical arithmetic certainly feature among skilled workers and were active in a broad range of numerically-intensive sectors, such as commerce, finance, surveying, and the mechanical arts. Although the number of cities is limited compared to the sample for which we have population data, when we replicate the staggered DiD event study with this outcome variable, we estimate a 16.65% increase in the real wages of skilled workers during the three decades following the adoption of practical arithmetic manuals (see figure B2).

Population in European cities based on Buringh's data. We replicate the previous analyses by considering only the Buringh (2021) dataset on European cities' population, without complementing it with Malanima's data on Italian cities. All results are confirmed, remaining qualitatively unchanged (see tables B10 and B11; figures B3, B4, and B5).

Matched samples. To compare entities with similar characteristics, we match cities with and without practical arithmetic manuals based on observables. As shown in tables B12 and B13, all results are confirmed.

Balanced panel in city-century regressions. As we are dealing with historical sources on city population data, the resulting city-century panel is unbalanced, with missing values concentrated in the older centuries. We replicate the panel fixed-effect analysis for cities that are observed throughout the entire period (14th–17th centuries). Results, reported in table B14, show a stronger, significant association between the presence of manuals and city population at the end of the century for all the manual variables.

Placebo test: Ptolemaic astronomy manuscripts. To rule out the academic knowledge channel, we perform a placebo test tracking the circulation of theoretical Indo-Arabic numerals via Latin astronomical manuscripts (proxied by the *Ptolemaeus Arabus et Latinus* corpus). Panel fixed-effect estimates reveal no relationship between the presence of these university-level texts and urban growth (see table B15).

Heterogeneous effects by city size. We test whether the results are driven by city size by creating a dummy variable that captures small cities. Table B16 shows that small cities benefit the most from the presence of practical arithmetic manuals.

Alternative controls. In pooled OLS estimates, we further control for city location and spatial dependency by replacing country fixed effects with the *longitude* of the city, its *latitude*, and *their interaction.* In panel fixed-effect analysis at the city-century level, we add country-x-century fixed effects and Black Death controls to account for events like country-specific wars or plagues. In the mechanism analysis, instead, we add controls for the *growth of notable people* and the *estimated population growth* between the two periods to better account for the increase in relative importance of the city. Results, reported in tables B17, B18, B19, B20, and B21 are qualitatively unchanged.

7 Conclusions

This paper explores the idea that the opportunities for growth depend on the socially distributed applications of knowledge to solving emergent problems and devising new tools and techniques, which in turn open new avenues for economic development and generate economic value. We have focused on the diffusion of a major mathematical innovation in commercial practices, its implications for knowledge production and development, and its effects on economic growth. With respect to other forms of science and technology, mathematics provides less immediate illustrations of discontinuities associated, for example, with industrial revolutions. Yet, mathematical tools share some of the characteristics identified in general-purpose technologies (Bresnahan and Trajtenberg 1995; Jovanovic and Rousseau 2005; Bresnahan 2010) and can indeed produce paradigmatic change (Dosi 1988) in the structure and evolution of economic systems. The statistical and data-science foundations of a variety of advanced digital technologies may come to mind. There are of course other examples. Borjas and Doran (2012), for instance, have studied the migration of Soviet mathematicians to North America, and the impact of their distinctive mathematical knowledge. In his path-breaking paper on the economics of scientific research, Nelson (1959) spoke about the importance of Maxwell's equation for technical progress.

The particular type of innovation we have studied in this paper—the introduction of Indo-Arabic numerals—was an instrument that provided a range of very practical uses for the institutions of modern markets, and can indeed be understood as the diffusion of a general-purpose technology. We argue that the socially distributed diffusion in Europe of this kind of mathematics and of the practical knowledge that was associated with it was indeed a numerical revolution, without which the prospects of economic growth would have been different. Since commerce is the first field in which practical arithmetic spread, our analysis has centered on commercial practices. However, as recently discussed by Kelly and Ó Gráda (2022), in time, practical arithmetic spread to a broad range of sectors, including surveying, navigation, instrument making, engineering, shipbuilding, architecture, and ballistics (see also Danna 2026).

The adoption of this particular body of knowledge was organized in society through purposeful learning activities in institutions, the practical arithmetic schools, which formed a vernacular curriculum parallel to that of the universities, and provided a key channel for the diffusion of mathematical innovations. It is important to stress that, as we have shown in sections 2 and 3, practical arithmetic texts reached, at different times for different places, a wide social circulation through a series of bottom-up decisions that did not follow academic hierarchies. While the tradition of the Latin *algorismi* was dominated by the names of a few authors,[25] the European tradition of practical arithmetic comprises hundreds of 'minor' authors who were mathematical practitioners active in the world of the mechanical arts. The social, temporal, and spatial diffusion of Indo-Arabic numerals among European practitioners unleashed the economic potential of the new mathematical techniques. These became a sort of 'social technology' (Nelson and Sampat 2001) for devising new ways to manage trade, make commercial decisions, design instruments, find routes at sea, improve precision, calculate proportions and exchanges, predict, survey, etc. One is left to wonder what type of market economy would have developed without this particular application of human intellect. This combination of mathematical techniques, their practical applications, and their social circulation constituted a crucial engine of epistemic, technical, and social changes, shedding light on the complex dynamics of 'modern' economic development.

25. Such as Alexander of Villedieu and Johannes de Sacrobosco.

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A Data appendix

For Online Publication

A.1 Database of practical arithmetic manuals: description and sources

Running from the late 13th century to 1600, the database of practical arithmetic manuals includes both manuscript and printed sources. Each manuscript source is considered in the dataset as an independent document, while printed manuals are recorded at the book-edition level. This choice is due to three independent reasons: (1) the different characteristics of manuscript and printed sources, (2) the varying social circulation of European practical arithmetic manuals, both manuscript and printed, and (3) the nature of the evidence extant. With regards to reason (1), while there is a degree of variation between copies of the same edition of a printed text (and this is particularly true for early incunabula), these differences are not comparable to the diversity presented by manuscript copies of the same text. This is due to the malleability of the manuscript medium in comparison with the standardization determined by movable-type printing. This is particularly true for practical sources such as practical arithmetic manuals, because manuscript practical texts were often not copied with the aim of reproducing the original text, but, rather, with the aim of meeting their users' individual needs (Bocchi 2017; Long 2009).

As it was common for manuscript manuals to be handed down across generations of users, manuscript manuals often show sections added by different hands at different moments in time. Just to mention a few examples, the earliest Venetian manual presents these characteristics, as it is a notebook written by several hands starting from the 1340s which covers arithmetical operations with fractions, a variety of commercial problems, a section on practical geometry, and one on commercial customs, weights, and measures.[1] With its 18 surviving manuscript copies, Maestro Benedetto's *Trattato d'abacho* (c. 1465) was probably the most influential manuscript abacus manual of the 15th century. These copies show a high degree of variation, as copyists selected the parts which were most interesting for them, and arranged

1. Florence, Biblioteca Riccardiana, ms. Ricc. 2161.

the sections according to their needs (Franci 2003; Ulivi 2002; Van Egmond 1980). A late example of the highly distinctive characteristics of each manuscript manual is the so-called memorandum-book of Richard Hill, a grocer from London who in the first half of the 16th century compiled a manuscript which mixes Latin, French, and English, gathering transcriptions of popular carols, chronicles, tables of exchange for wool cloth across the Channel, personal notes, and a text on practical arithmetic.[2] This high specificity of each manuscript manual justifies the choice to consider copies of the same text as independent items in the database.

With regards to the social circulation of these texts—i.e. reason (2)—the 'social reach' of each practical arithmetic manual tended to decrease with time. As we have seen, early manuscript manuals were probably texts that circulated in the hands of practical arithmetic masters. This makes these texts distinctively public sources, which were transmitted across generations of masters, and whose mathematical contents 'reached' hundreds of students. This is not limited to the Italian tradition of abacus mathematics. One of the earliest documents attesting a French-vernacular tradition of practical arithmetic is a manuscript written by two early practical arithmetic masters active in the south of France in the 1470s, Barthélemy de Romans and Mathieu Préhoude. The latter was possibly one of the first practical arithmetic masters in Lyon, and the former's pupil (Spiesser 2000, 2003; Hay 1988).[3] One of the first texts to attest the knowledge of algebra in Germany is an anonymous manuscript written in 1481 which was used by several generations of German *Rechenmeister* (Vogel 1981; Folkerts 2002).[4]

With time, there is an increasing number of manuscript manuals written or owned by scribes who were not masters. For example, we have evidence of 15th-century manuals written by a rag trader (1464)[5] and by a painter (c. 1480),[6] as well as of manuscripts owned by a

2. Oxford, Balliol College, ms. 354.
3. Cesena, Biblioteca Malatestiana, ms S-XXVI-6.
4. Dresden, Sächsische Landesbibliothek, Codex Dresdensis C 80.
5. Bologna, Biblioteca Universitaria, ms. 1612.
6. Florence, Biblioteca Mediceo-Laurenziana, ms Ash 359 (291).

carpenter (1479),[7] and by a medical practitioner (c. 1460).[8] This is again not limited to the Italian tradition, as there is evidence that manuscript manuals were circulating among Catalan merchants from the late 15th century (Rey 2004).[9] As the first printed practical arithmetic manual was published in 1478,[10] and as the production of printed practical arithmetic manuals did not take off until the last decade of the 15th century, this increasing social circulation of practical arithmetic texts predated the effects on this tradition of the introduction of printing, which, as a consequence, acted as a rapid accelerator of a pre-existing trend. Thanks to much lower prices, printed manuals of practical arithmetic became affordable to a wider public, and therefore increasingly circulated in the hands of practitioners and students. While the printing press dramatically expanded the number of copies in circulation, the 'social reach' of practical arithmetic manuals decreased, as they moved from being public texts (in the case of early manuscript manuals used to lecture in front of generations of students) to increasingly private documents (e.g. printed manuals used by a single individual).

The third reason to record printed manuals at the book-edition level relates to the evidence extant. While we have good evidence regarding the editions of early printed European texts, with over 27,000 editions known before 1501, we have details about their print runs in only 160 cases. Moreover, these are often editions with special characteristics, making this evidence exposed to selection bias (Nuovo 2013, 104). As a consequence, the widespread scholarly practice is to set the focus of analysis to the book-edition level, and to consider the number of reprints as a proxy for the success of an early modern printed text. For example, scholarly catalogs, both printed and digital, typically include information at the book-edition level (Smith 1908, 1939; Van Egmond 1980; Hoock et al. 1991; Navarro Brotons 2000). For all these reasons concerning the different characteristics of manuscript and printed texts, the social circulation of practical arithmetic texts, and the available evidence on printed editions, considering manuscript documents at the copy-level and printed documents at the

7. Florence, Biblioteca Marucelliana, ms. A. c. s. 47.
8. Florence, Biblioteca Mediceo-Laurenziana, ms. Ash 1128.
9. Arxiu Históric de Mallorca, ms Diversos 37B/2. Antigua signatura: C108.
10. This is the anonymous *Aritmetica di Treviso*.

edition-level seemed the preferable choice.

Both secondary and primary sources were used in the construction of the database of practical arithmetic manuals. Catalogs are the most important secondary source. With respect to the Italian abacus tradition, the database includes all the evidence from the catalog edited by Van Egmond (1980), which remains the most comprehensive source of its kind documenting this tradition. Even though this catalog is relatively more thorough on manuscript sources than printed books (it does not provide summaries of contents for the latter), whenever possible, these sources were consulted directly, either through direct inspection or through digitized copies, and their content was manually recorded. Moreover, several findings of successive research in the field were integrated in the evidence provided by Van Egmond (Bocchi 2017; Franci 2015, 2003; Ulivi 2011, 2002; Long 2009).

With respect to the European tradition, the database includes a variety of sources. Their foundation is the study by Smith (1908), a monumental work based on extensive archival research efforts. This source needed to be treated with care and required the definition of precise criteria since it also includes texts that are arguably not part of a European tradition of practical arithmetic. Firstly, our database excludes all texts that do not present Indo-Arabic numerals. Secondly, among the texts that do use Indo-Arabic numerals, we retained only those that are part of the practical arithmetic tradition. This implies the exclusion of the early modern reprints of classical, as well as early medieval, sources (e.g. all the reprints of Boethius' Arithmetica and all the early modern texts in this tradition, jointly with works related to numerology and the computi for the calculation of the calendar).[11] Thirdly, the database includes only texts that were relevant for practice in that they 1) contained practical applications of mathematics; 2) were explicitly addressed to practitioners, or 3) could have been used in activities of commercial training. The implication is that purely theoretical works developed in universities (e.g. most of the algorismi tradition) did not enter

11. The computi were texts that taught how to calculate the calendar and were widely used, particularly in ecclesiastical contexts, for the calculation of important dates, such as Easter. They constituted a relevant channel of diffusion for Indo-Arabic numerals and in a number of cases they preceded by a considerable margin the spread of practical arithmetic (Nothaft 2014).

the database. Theoretically-oriented algebraic texts were only included if they contained at least some practical applications.[12]

Relevant online repositories, depending on the kind of text under investigation, were used to check Smith's records. These included: the *Incunabula Short Title Catalogue* (ISTC), the *Universal Short Title Catalogue* (USTC), the *English Short Title Catalogue* (ESTC), the online catalog of the Bibliothèque Nationale de France and their digitalization project (Gallica), the Münchener DigitalisierungsZentrum of the Bayerische Staatsbibliothek, and the Biblioteca Virtual Miguel de Cervantes (Cervantes Virtual). These research tools, which were unavailable to both Smith and Van Egmond, allowed for a significant expansion of the information provided on every text, including identification of new texts by the same author, identification of new authors, and discovery of 1) editions not included in older catalogs, 2) holding institutions, and 3) classmarks of original copies. Moreover, it was possible to identify and report every digital reproduction. When referring to these research tools, the risk of circularity was avoided by including their additional evidence only if it was based on information richer or independent from the sources that were already being consulted.

The research protocol we have just described (i.e. the criteria for inclusion in the database and the use of online repositories) was applied to the consultation of all catalogs. The texts cited in Smith (1939) were included only if it was clear that they had been consulted by Smith himself or if the USTC provided expanded information on them, therefore corroborating Smith's information. Navarro Brotons (2000) contained valuable information on Iberian sources, whereas Hoock et al. (1991) helped to consolidate the evidence available for central and northern Europe. The information provided by these sources was integrated by means of an extensive list of specialistic papers and local studies.

12. Among the relevant examples, the De arte supputandi libri quattuor by Cuthbert Tonstall (London, 1522) is included in the database despite its predominant theoretical focus because it is the first text published in England that was entirely dedicated to arithmetic. Similarly, Petrus Ramus' Arithmeticae libri tres (first ed. Paris, 1555) is included because of the importance attributed by the author to applied mathematics (Angelini 2008). Among the texts that are excluded, even though they were of demonstrable practical relevance, are the manuals concerning finger reckoning. Some of the late manuals on finger reckoning present Indo-Arabic numerals, but belong to a different tradition than that of practical arithmetic.

Moreover, one of the authors of this paper personally visited archives in Florence and in Bologna, as well as archives in the British Library in London, the Cambridge University Library in Cambridge (UK) and the Bibliothèque nationale de France in Paris. The visits generated first-hand evidence and new documentation on early primary sources—of great interest—of Italian and European practical arithmetic traditions. Whenever possible, during the visits, the historian inspected the content of the texts. Through a combination of direct inspection of original or digitized copies, and through secondary studies, this work has generated data on the contents of 1051 texts (82% of all recorded texts).

Figure A1 provides an example of an unpublished 14th-century manual of practical arithmetic from Florence. The folios represent, respectively, a) an introductory page providing conversion tables between monetary units and a table of contents; b) multiplication tables; c) a problem asking to calculate the duration of the same maritime voyage using different types of sails. These examples are taken from one of the most important abacus manuals written in 14th-century Florence: Paolo dell'Abaco's *Trattato di tutta l'arte dell'abacho*, written around 1339. Figure A2 gives an example of a printed manual of practical arithmetic, provided by Adam Ries' *Rechenung auff der linihen und federn* (1527). With over one hundred reprints, this was the most successful practical arithmetic manual of 16th-century Germany. The image reproduces, respectively, a) the title page of the manual; b) the page introducing Indo-Arabic numerals; c) the section dedicated to problems of exchange. How representative is this sample of texts? We cannot provide a definitive answer to this question because the size of the population of reference is unknown. It is extremely difficult to estimate the sample's representativeness, considering that we do not know how many manuals of practical arithmetic were written in Europe between the 13th and the end of the 16th century. We do know, however, that the Italian abacus tradition is reasonably well documented during the first part of the observation period (14th and early 15th centuries). The same cannot be said for other parts of Europe, e.g. Spain and the south of France, where there is evidence of far fewer manuscripts than in the case of Italy. As we have already mentioned, we do

Figure A1: Example of a 14th-century Florentine manual of practical arithmetic: Paolo dell'Abaco's *Trattato di tutta l'arte dell'abacho* (c. 1339). Source: Florence, Biblioteca Nazionale Centrale, ms. Fondo Nazionale, II, IX, 57, fols. 19v, 25v, 121v.

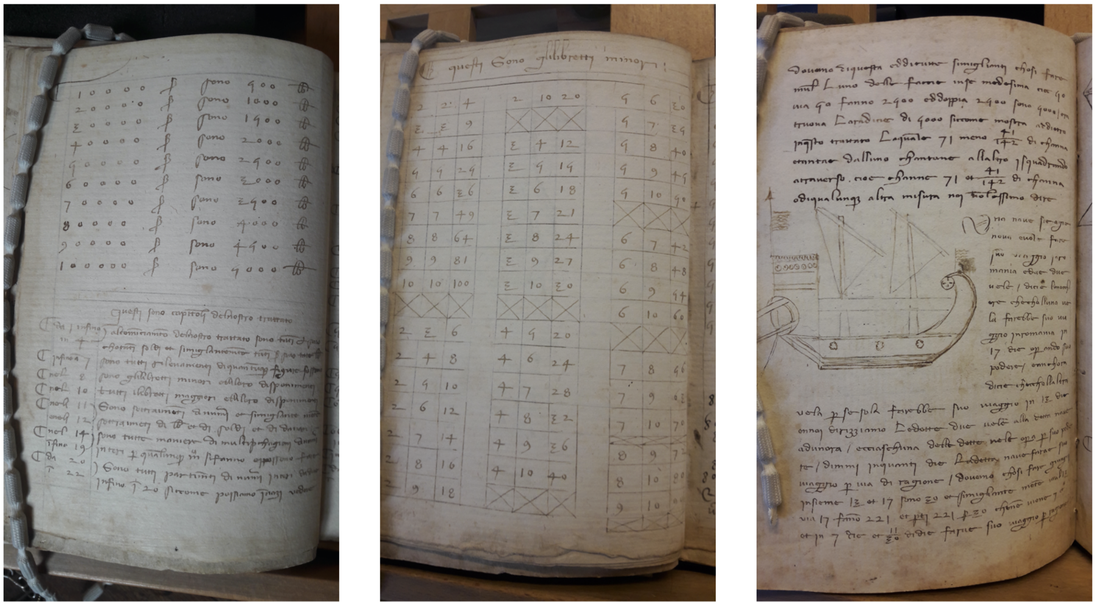
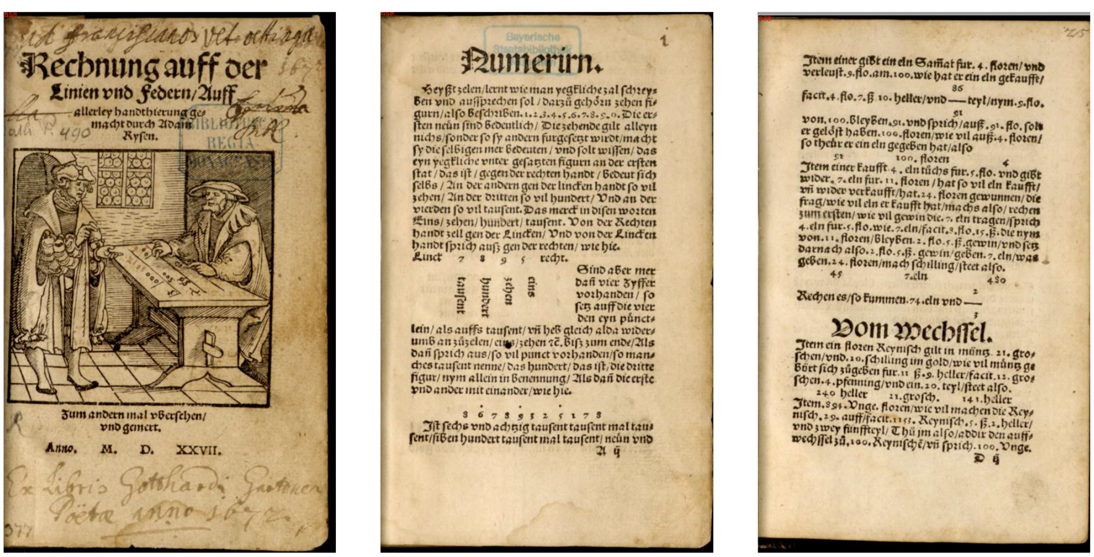


Figure A2: Example of a printed practical arithmetic manual: Adam Ries (1527) *Rechenung auff der linihen und federn*, fols. ir, 1r, 25r. Image: Bayerische Staatsbibliothek, M unchener DigitalisierungsZentrum



not have a good picture of the mathematical exchanges that may have taken place in the early phase in the Mediterranean region because several sources may not have survived to the present day. It is possible that the manuscript material of the very early stages of the diffusion of European practical mathematics may be lost, except for relevant Italian sources. Of the known surviving manuscripts from the pre-printing age, our database offers very good coverage.

The quantity and quality of available sources increase considerably as we enter the age of the printing press. An overwhelming amount of the texts was produced in this period. Although it is conceivable that some materials are missing, the most reliable repositories currently available were used to compile detailed information on these stages of diffusion, and whenever possible, each and every record has been manually double-checked.[13] The sources concerning the printing-press-era manuscripts from outside of Italy have not been studied as comprehensively as would be desirable.

Nevertheless, it is important to emphasize that (1) known sources of European printed texts can probably be considered as a representative sample of their respective traditions,[14] and (2) the database provides a highly satisfactory coverage of them. Crucially, the quality of the data is even better for Italy, for which the data can be considered a reliable sample of both the manuscript and printed traditions.

To summarize, in the case of Italy, both the manuscript and printed traditions have been studied and are relatively well preserved. The database offers good coverage of both. As far as the broader European region is concerned, few early manuscripts have been preserved, but the known sources have been studied in detail and recorded in the database. Conversely, manuscript material from the printing-press era has not been comprehensively studied and, as a consequence, is probably not recorded in full. This, however, does not represent a concern, as evidence on the circulation of practical arithmetic at the city-level in this period is provided by printed sources. Printed European material is instead abundant: it can be

13. The database relies on sources that cover most Western European areas. The data collected is arguably stronger for Italian, German, and English printed texts, given the amount of studies that have addressed the areas (especially Italy) and the quality of the work that has been done on cataloging and digitizing primary sources (especially for Germany and England).

14. Information on editions of early European printed texts is abundant, as we know of over 27,000 editions of printed texts between 1450 and 1500 (Nuovo 2013, 104).

considered qualitatively and quantitatively representative of its tradition, and has been thoroughly recorded in the database. Given the type of historical sources, the database contains as good a sample as possible of the known sources, and one that is arguably representative for our study.

A.2 Historical evidence on the diffusion of practical arithmetic manuals in Europe

Practical arithmetic manuals and practical arithmetic schools appeared in Italian city states between the late 13th and the 14th centuries, as is discussed in 2.2. In other European areas, these manuals emerged in later periods. In France, an early vernacular manual (1470s) was written by one of the first practical arithmetic masters active in Lyon, Barthélemy de Romans (Spiesser 2000). De Romans was soon followed by Nicholas Chuquet, the author of the important *Triparty en la science des nombres* (1484), who consolidated the tradition of Lyonnaise practical arithmetic schools (Benoit 1988; Flegg et al. 1985). These first manuscript texts were followed by a series of printed manuals, which grew steadily as Lyon became a key hub of European finance and trade. In Germany, the first manuscripts of practical arithmetic were written in the 1450s in Regensburg, under the influence of Venetian sources (Vogel 1954). These were followed by the first printed vernacular manuals (the *Rechenbücher*), published in Bamberg by Ulrich Wagner in 1482-1483 (Vogel 1980; Schröder 1996).[15] Following these first texts, the tradition of German *Rechenbücher* grew exponentially in subsequent decades together with the spread of the *Rechenschulen*, which provided the foundational training for generations of German merchants (Gebhardt and Albrecht 1996; Denzel 2002). Following the publication of *Die maniere om te leeren cyffren* in Brussels (1508) and in Antwerp (1510), the manuscript by the *rekenmeester* Christianus van Varenbraken (c. 1532) documents that also in the Low Countries the publication of these manuals was coupled with the foundation of practical arithmetic schools (Kool 1988; Gärtner 2000, 244–47). These

15. Wagner was a practical arithmetic master (*Rechenmeister*) who ran a school in Nuremberg in the second half of the 15th century.

schools flourished first in Antwerp and, subsequently, in Amsterdam (Meskens 2013). After the first manuals printed in the 1520s and 1530s under the influence of French and Dutch texts, the English tradition of practical arithmetic manuals took off in 1543 with the publication of Robert Recorde's *The Ground of Artes* (Williams 2012). The popularity of these texts grew together with the foundation of practical arithmetic schools in London by the first generation of English masters (Woodbridge 2003, 3–5).

This chronology shows that data on practical arithmetic manuals does not provide a measure for economic activity in general, but rather a proxy for the spread of specific commercial skills. If this was not the case, we would observe practical arithmetic manuals with Indo-Arabic numerals appear simultaneously in the main European trading and industrial centers of the 13th and 14th centuries—i.e. not only in Italy and southern Europe, but also in the Low Countries, Hansa cities, and England.

For example, both Florence and Bruges were key economic centers in 14th-century Europe, as both cities had a flourishing wool and cloth industry and relied on an extensive trade network (Goldthwaite 2009; Murray 2009). However, while the 14th century was the heyday of Florentine abacus mathematics—as marked by a booming production of manuscript manuals—in Bruges the first practical arithmetic manuals with Indo-Arabic numerals were published only in the second half of the 16th century.[16] Significantly, this late appearance of vernacular practical arithmetic manuals corresponds with the fact that international finance in 14th-century Bruges was dominated by Florentines, and with the late adoption among Bruges merchants of the business practices of the commercial revolution (De Roover 1948).[17]

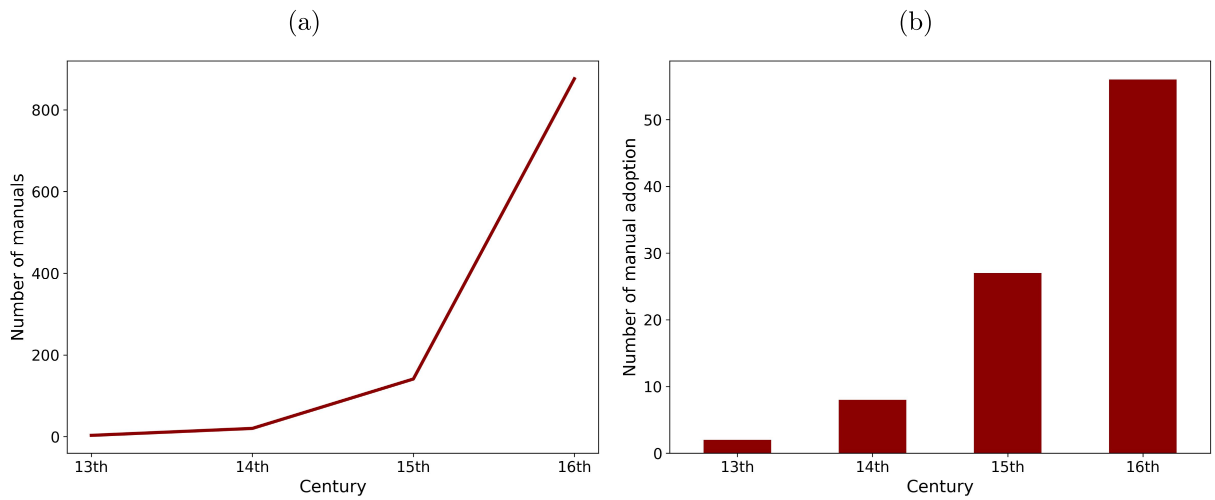
16. The first practical arithmetic manual published in Bruges was probably Adriaen van der Gucht's *Cijfer bouck, inhoudende vele nieuwe, fraye, ende gherievighe practijcken van arithmetica*, published in 1569.

17. Similar considerations can be made about other important trade centers of northern Europe. Lübeck, for example, despite being among the leading Hanseatic cities in the 14th century—with its port providing a hub for maritime trade across the North Sea—had its first practical arithmetic manuals published only in the second half of the 16th century.In other prominent Hanseatic commercial cities, such as Bremen and Lüneburg, no practical arithmetic manuals with Indo-Arabic numerals were published before 1600. England was also a late adopter of Indo-Arabic numerals in commercial practices, as the first manuals of practical arithmetic in England were published in London in the 1530s (Williams 2012). It may not be by chance that, until the late 15th century, calculations of exchange proved 'of the greatest difficulty' for English merchants active in international trade (Hanham 1985, 165–66).

A.3 Additional descriptive statistics

City-century data. Figure A3 reports the diffusion, in terms of the number of manuals and the number of new cities that adopt those manuals, of practical mathematics during the period of interest. Combined with the information provided in table 1, this figure shows that for the period under investigation, the spread of practical arithmetic manuals in Europe started in the 13th century with 3 manuals in 2 Italian cities and ended in the 16th century with 876 manuals in 82 cities in twelve European countries.

Figure A3: Number of practical arithmetic manuals and number of cities adopting manuals per century.



Tables A1 and A2 report the descriptive statistics and correlation tables of the variables used in the empirical analyses.

Table A1: Descriptive statistics for century data.

(a) Continuous variables.

Variable	Mean	Std	Min	Median	Max	Count
Century data						
Nb. of manuals (t-1)	0.58	5.04	0	0	101	1,799
Nb. of manuals 2nd half century (t-1)	0.38	3.49	0	0	67	1,799
Beg-of-century population (in thousands)	11.34	17.57	1	6	300	1,799
End-of-century population (in thousands)	13.29	27.18	1	7	575	1,799

(b) Binary variables.

Variable	N	%	Count
Century data			
Manuals dummy (t-1)	127	7.06	1,799
Printing press (t-1)	457	25.46	1,799
University	225	12.51	1,799
Capital	135	7.50	1,799
Roman road	981	54.59	1,799
Navigable river	880	48.92	1,799
Mediterranean port	181	10.06	1,799
Atlantic port	27	1.50	1,799
Baltic port	27	1.50	1,799
North Sea port	112	6.23	1,799

Table A2: Correlation matrix on century data.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1: Beg-of-century population	1.00	0.86	0.46	0.44	0.44	0.28	0.30	0.46	0.18	0.13	0.10	0.11	0.02	0.01
2. End-of-century population	0.86	1.00	0.47	0.49	0.39	0.27	0.24	0.45	0.14	0.13	0.07	0.10	0.03	0.07
3. Nb. of manuals (t-1)	0.46	0.47	1.00	0.97	0.42	0.19	0.10	0.16	0.02	0.07	0.02	-0.00	-0.00	0.02
4. Nb. of manuals 2nd half (t-1)	0.44	0.49	0.97	1.00	0.40	0.18	0.09	0.15	0.02	0.07	0.01	-0.01	0.00	0.03
5: Manuals dummy (t-1)	0.44	0.39	0.42	0.40	1.00	0.40	0.26	0.23	0.09	0.15	0.00	0.04	0.02	0.00
6: Printing press (t-1)	0.28	0.27	0.19	0.18	0.40	1.00	0.31	0.14	0.12	0.23	-0.03	0.03	0.02	-0.01
7: University	0.30	0.24	0.10	0.09	0.26	0.31	1.00	0.23	0.18	0.16	0.04	-0.02	0.05	-0.07
8: Capital	0.46	0.45	0.16	0.15	0.23	0.14	0.23	1.00	0.10	0.10	0.09	0.03	0.02	-0.03
9: Roman road	0.18	0.14	0.02	0.02	0.09	0.12	0.18	0.10	1.00	0.09	0.20	0.06	-0.14	-0.12
10: Navigable river	0.13	0.13	0.07	0.07	0.15	0.23	0.16	0.10	0.09	1.00	-0.28	0.02	0.03	0.03
11: Mediterranean port	0.10	0.07	0.02	0.01	0.00	-0.03	0.04	0.09	0.20	-0.28	1.00	-0.04	-0.04	-0.09
12: Atlantic port	0.11	0.10	-0.00	-0.01	0.04	0.03	-0.02	0.03	0.06	0.02	-0.04	1.00	-0.02	-0.03
13: Baltic port	0.02	0.03	-0.00	0.00	0.02	0.02	0.05	0.02	-0.14	0.03	-0.04	-0.02	1.00	-0.03
14: North Sea port	0.01	0.07	0.02	0.03	0.00	-0.01	-0.07	-0.03	-0.12	0.03	-0.09	-0.03	-0.03	1.00

20-year interval data. Figure A4 reports the number of new cities that adopt practical arithmetic manuals during the period of interest. Tables A3 and A4 report the descriptive statistics and correlation tables of the variables used in the empirical analyses.

Figure A4: Number of cities adopting manuals per 20-year interval.

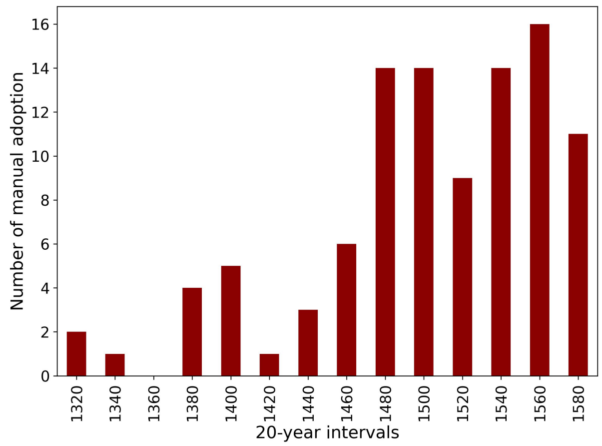


Table A3: Descriptive statistics for 20-year interval data.

(a) Continuous variables.

Variable	Mean	Std	Min	Median	Max	Count
20-year interval data						
Nb. of manuals	0.09	1.05	0	0	33	11,578
Nb. of merchants	0.01	0.17	0	0	6	11,578
Nb. of merchants (t+1)	0.02	0.18	0	0	6	11,578
Nb. of notable people	1.14	4.01	0	0	92	11,578
Nb. of *Ars Mercatoria* texts	0.22	1.77	0	0	59	9,097
Nb. of *Ars Mercatoria* texts (t+1)	0.22	1.77	0	0	59	9,097
Nb. of printed books	35.53	330.57	0	0	11265	9,097

(b) Binary variables.

Variable	N	%	Count
20-year interval data			
Manuals dummy	270	2.33	11,578
Merchants dummy (t+1)	144	1.24	11,578
Ars Mercatoria texts dummy (t+1)	458	5.03	9,097
Printing press	1760	15.20	11,578
University	628	5.42	11,578
Capital	486	4.20	11,578

Table A4: Correlation matrix on 20-year-interval data.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1: Nb. of merchants	1.00	0.48	0.33	-	-	-	0.32	0.29	0.47	0.12	0.06	0.15	-
2: Nb. of merchants (t+1)	0.48	1.00	0.85	-	-	-	0.27	0.28	0.43	0.12	0.06	0.14	-
3: Merchants dummy (t+1)	0.33	0.85	1.00	-	-	-	0.23	0.25	0.37	0.13	0.06	0.15	-
4: Nb. of *Ars Mercatoria* texts	-	-	-	1.00	0.72	0.38	0.56	0.33	-	-	0.07	0.21	0.47
5: Nb. of *Ars Mercatoria* texts (t+1)	-	-	-	0.72	1.00	0.53	0.54	0.35	-	-	0.06	0.20	0.49
6: *Ars Mercatoria* texts dummy (t+1)	-	-	-	0.38	0.53	1.00	0.34	0.42	-	-	0.24	0.24	0.35
7: Nb. of manuals	0.32	0.27	0.23	0.56	0.54	0.34	1.00	0.56	0.52	0.19	0.12	0.18	0.73
8: Manuals dummy	0.29	0.28	0.25	0.33	0.35	0.42	0.56	1.00	0.52	0.30	0.25	0.25	0.48
9: Nb. of notable people	0.47	0.43	0.37	-	-	-	0.52	0.52	1.00	0.37	0.27	0.43	-
10: Printing press	0.12	0.12	0.13	-	-	-	0.19	0.30	0.37	1.00	0.29	0.19	-
11: University	0.06	0.06	0.06	0.07	0.06	0.24	0.12	0.25	0.27	0.29	1.00	0.31	0.13
12: Capital	0.15	0.14	0.15	0.21	0.20	0.24	0.18	0.25	0.43	0.19	0.31	1.00	0.20
13: Nb. of printed books	-	-	-	0.47	0.49	0.35	0.73	0.48	-	-	0.13	0.20	1.00

B Additional results

For Online Publication

B.1 *Ars Mercatoria* texts

To analyze the association between practical arithmetic manuals and *Ars Mercatoria* texts, we estimate the following models via OLS regressions:

$$\text{Ars Mercatoria texts}_{c;\,t+1} = \beta_0 + \beta_1\,\text{Manuals}_{c;\,t} + \gamma\mathbf{X}_{c;\,t} + \delta_t + \phi_c + \epsilon_{c;\,t}, \tag{5}$$

where c is a city, t is the 20-year interval, Ars Mercatoria $\text{texts}_{c;\,t+1}$ indicates, alternatively, the log-transformed number of *Ars Mercatoria* texts at time t+1 and and the corresponding dummy variable, $\text{Manuals}_{c;\,t}$ indicates the main independent variables (i.e., the log-transformed number of manuals and the manual dummy), $\mathbf{X}_{c;\,t}$ is a vector collecting the set of controls at the city-20-year-interval level, δ_t and ϕ_c capture, respectively, time and city fixed effect, and $\epsilon_{c;\,t}$ is the error term. All estimations report clustered standard errors at the city level. The set of controls includes the presence of a university or state capital and the log-transformed number of *Ars Mercatoria* texts in the 20-year interval. As the *Ars Mercatoria* texts are all printed books, we control for the log-transformed number of printed books at time t. Alternative specifications control for the growth of notable people between the two periods as a proxy for changes in city prestige and the estimated population growth (based on the century population growth–see table B21).

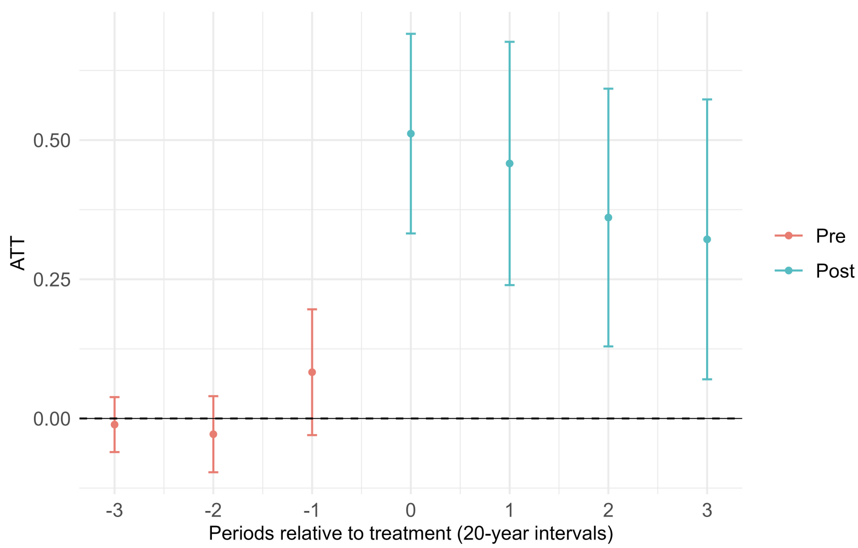
Results are reported in table B1. Results of the event study where the outcome variable is the logarithm of the number of *Ars Mercatoria* texts plus 1 in the city in an interval of 20 years, and a control group composed of not-yet-treated cities are shown in figure B1.

Table B1: Practical arithmetic manuals and the number of *Ars Mercatoria* texts in the following 20 years.

Dependent Variables:	log(Nb. of AM texts$_{t+1}$+1)		AM text dummy$_{t+1}$	
	(1)	(2)	(3)	(4)
log(Nb. of manuals$_t$+1)	0.205***		0.062***	
	(0.031)		(0.021)	
Manuals dummy$_t$		0.129***		0.088***
		(0.042)		(0.029)
University	0.024	0.024	0.054**	0.054**
	(0.022)	(0.023)	(0.023)	(0.023)
Capital	0.036	0.022	-0.003	-0.005
	(0.033)	(0.033)	(0.025)	(0.024)
log(Nb. of printed books$_t$+1)	0.036***	0.045***	0.032***	0.032***
	(0.005)	(0.005)	(0.004)	(0.004)
log(Nb. of *Ars Mercatoria* texts$_t$+1)	0.348***	0.385***	0.161***	0.165***
	(0.031)	(0.032)	(0.017)	(0.017)
City FE	Yes	Yes	Yes	Yes
20y interval FE	Yes	Yes	Yes	Yes
Observations	9,097	9,097	9,097	9,097
R^2	0.630	0.623	0.483	0.483
Within R^2	0.247	0.233	0.122	0.123

Notes: This table presents panel fixed-effect OLS estimates of equation 5. The dependent variables are the logarithm of the number of *Ars Mercatoria* texts + 1 at time $t+1$ and a dummy variable equal to one if there is at least one *Ars Mercatoria* text at time $t+1$. t indicates the 20-year intervals after the emergence of the printing press. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city at $t-1$. Control variables include the presence of a university or a state capital. We also control for the number of printed books at time t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Figure B1: Event study on the number of *Ars Mercatoria* texts printed in 20 years.



Notes: The figure shows DiD coefficients estimated following equation 3 for European cities. Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of the number of *Ars Mercatoria* texts plus 1 in the 20-year interval. Treatment occurs at time 0. Pre-treatment is conditional on the number of printed books in the city. P-value for pre-test of parallel trends assumption (Ward test): 0.23. Confidence intervals are 95%. Standard errors are clustered at the city level.

B.2 Instrumental variable approach.

Table B2: Centers of diffusion of practical arithmetic manuals across linguistic areas.

City	Linguistic Area	Century
Perugia	Italy	1200
Sevilla	Spain	1300
Bamberg	Germany	1400
Lyon	France	1400
Valencia	Catalonia	1400
Bruxelles	Low Countries	1500
Debrecen	Hungary	1500
Kobenhavn	Denmark	1500
Krakow	Poland	1500
Lisboa	Portugal	1500
London	England	1500

Notes: The table reports the city and linguistic area of the first occurrence of practical arithmetic manuals written in local vernacular in each linguistic area.

We define our instrumental variable as the distance between the city under investigation and the city of the same linguistic area where a practical arithmetic manual in the local vernacular appeared first. If no manual had yet appeared in a linguistic area in the century under investigation, the instrumental variable is equal to the minimum distance from the already existing centers of diffusion (see table B2). Since we are in a panel setting and practical arithmetic manuals spread across linguistic areas in different centuries, this instrument varies over time, following the diffusion of practical arithmetic manuals from the south to the north of Europe. Specifically, for the 13th century, the instrument is the distance between the focal city and Perugia (i.e., the center of diffusion for manuals in Italian). With the emergence of a new center of diffusion in the 14th century (i.e., Seville), the instrument returns the minimum value between the distance of the focal city from Perugia and the distance of the focal city from Seville. Considering the emergence of new centers of diffusion, an analogous algorithm is applied for the following centuries. By employing this instrument in panel fixed-effect two-stage least square (2SLS) estimations, we exploit the variation (reduction) of the distance from practical arithmetic manuals in the local language to explain the probability of observing a manual in the city under investigation.

Variations in the distance from the first center of diffusion in each linguistic area are exogenous to the determinants of urban growth. Indeed, the first appearance of manuals was not directly linked to economic trends in general but to the appearance of the specific business practices of the commercial revolution. Late medieval Europe had at least two areas characterized by considerable economic dynamism: the city-states of central-northern Italy and the cities of the Hanseatic League. Initially, practical arithmetic manuals only circulated in Italian city-states, where the commercial revolution originated. Hansa cities, on the contrary, were late adopters of this mathematics. This corresponds with their late adoption of the financial and accounting techniques of the commercial revolution, confirming the reading of practical arithmetic manuals as a good proxy for the techniques of the commercial revolution. Moreover, the cities in which practical arithmetic manuals first appeared were not necessarily the main economic centers of their regions. The Italian first city of diffusion—Perugia—was a more peripheral town than Pisa, which in turn was by far not the biggest city in its region in the 13th century (Florence, Naples, Venice, and Milan were at least twice as big as Pisa). Bamberg—the first city for the German language—had a declining population between 1300 and 1500, and Debrecen—the first city for the Hungarian language—was smaller than Pest and Buda in the 16th century. In several cases, including Seville, cities did not display superior growth performance before the appearance of manuals, but as manuals translated into local vernacular appeared in these cities, they became epicenters of the regional diffusion of this practical mathematics. As the diffusion dynamics of practical arithmetic manuals are independent of the dynamics of economic growth in Europe, the distance from the first center of diffusion in each linguistic area is a good candidate to instrument the number of practical arithmetic manuals in our exercise.[18] Therefore, we

18. Let us consider, for instance, German cities. In the 13th century, they were relatively close to one of the main economic centers of that time, i.e., Hansa cities, but quite far from Perugia (the only diffusion center of manuals in that century). While the closest main economic center (Hansa cities) remains the same over centuries, the value of the instrument for German cities diminishes over centuries, following the spread of

Table B3: Instrumental variable estimates: First stage.

	log(Nb. of manuals$_{t-1}$+1) (1)	log(Nb. of manuals 2^{nd} half-cent$_{t-1}$+1) (2)	Manuals dummy$_{t-1}$ (3)
log(Distance from 1^{st} man$_{t-1}$+1)	-0.124***	-0.104***	-0.068***
	(0.038)	(0.039)	(0.012)
Printing press$_{t-1}$	0.248***	0.218***	0.173***
	(0.046)	(0.041)	(0.027)
University	-0.042	-0.072	-0.032
	(0.099)	(0.078)	(0.047)
Capital	-0.095	-0.038	-0.080
	(0.104)	(0.102)	(0.087)
log(Beginning-of-century population$_t$)	0.095***	0.091***	0.036**
	(0.031)	(0.028)	(0.016)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.562	0.547	0.568
Within R^2	0.091	0.091	0.105

Notes: This table presents panel fixed-effect OLS estimates on European cities in the 14^{th}–17^{th} centuries. Dependent variables are the logarithm of the number of practical arithmetic manuals in the city in century $t-1$, the logarithm of the number of manuals in the second half of the century, and a dummy variable which signals the presence of manuals in century $t-1$. *Distance from 1st manual* refers to the distance of the city from the location of the first occurrence of practical arithmetic manuals in the local vernacular. Control variables include the presence in the city of printing presses in century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of the century t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

estimate a panel fixed-effect 2SLS regression for the three different indicators of the presence of manuals. The first-stage results (table B3) show that the distance from the first manual in the area is significantly and negatively correlated to the presence and the number of manuals in the city, confirming the importance of regional centers for the diffusion of these texts across Europe. As shown in the following analysis, we do not observe the same patterns when we consider—instead of the distance from cities in which practical arithmetic manuals appeared first—the distance from a random selection of the linguistic area's main economic

practical arithmetic manuals from the south to the north of Europe and the appearance of the first German practical arithmetic manual in Bamberg.

centers. This exercise provides additional evidence on the independence of the diffusion of practical arithmetic manuals from economic determinants, supporting the validity of our instrument. Moreover, the high values of F tests for the IV indicate that this distance is a strong instrument. The occurrence of practical arithmetic manuals is also associated with the presence of the printing press since several manuals, especially later ones, were printed texts. Table B4 reports the results of the second stage, which confirms the findings reported in table 2.

Falsification tests. We consider the distance of a city from large centers in the same linguistic area in the previous century as a placebo IV. We define large centers as cities in the top 5% of the population distribution of the century. We then randomly select a large center for every linguistic area in each century and use the distance from that center as the IV for the cities in the linguistic area. If there are no large centers in the linguistic area, we compute the distance from the closest large center of other linguistic areas. Table B5 reports the first stage of this falsification test on the IV and shows that the distance from the large cities of the linguistic area is not significantly correlated with the number of practical arithmetic manuals in the city.

Table B4: Instrumental variable estimates: Practical arithmetic manuals and the city population at the end of the following century.

Dependent Variable:	log(End-of-century population$_t$)		
	(1)	(2)	(3)
log(Nb. of manuals$_{t-1}$+1)	0.470***		
	(0.158)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.558***	
		(0.198)	
Manuals dummy$_{t-1}$			0.862***
			(0.316)
Printing press$_{t-1}$	-0.074	-0.079	-0.106
	(0.063)	(0.067)	(0.072)
University	0.015	0.035	0.023
	(0.080)	(0.078)	(0.076)
Capital	0.323***	0.300***	0.348***
	(0.107)	(0.102)	(0.129)
log(Beginning-of-century population$_t$)	0.247***	0.242***	0.261***
	(0.038)	(0.039)	(0.038)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.878	0.879	0.874
F-test (1^{st} stage)	76.5	72.4	76.1

Notes: This table presents panel fixed-effect 2SLS estimates of the following model: $\log(\text{End-of-century pop}_{c;\,t}) = \beta_0 + \beta_1 \,\text{Manuals}_{c;\,t-1} + \gamma \mathbf{X}_{c;\,t} + \delta_t + \phi_a + \epsilon_{c;\,t}$. The dependent variable is the logarithm of the population level at the end of century t. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. We instrument these variables with the city's distance from the location of the first occurrence of practical arithmetic manuals in the local vernacular. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of city population at the beginning of the century t and add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B5: IV falsification test: First stage.

	log(Nb. of man_{t-1}+1) (1)	log(Nb. of man 2^{nd} half-century_{t-1}+1) (2)	Manuals dummy_{t-1} (3)
log(Distance from large city_t-1$+1)	0.015	0.010	-0.013
	(0.066)	(0.051)	(0.021)
Printing press_{t-1}	0.284***	0.249***	0.192***
	(0.045)	(0.041)	(0.026)
University	-0.026	-0.059	-0.025
	(0.097)	(0.076)	(0.046)
Capital	-0.118	-0.057	-0.092
	(0.108)	(0.099)	(0.091)
log(Beginning-of-century population_t)	0.120***	0.111***	0.049***
	(0.036)	(0.034)	(0.018)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.544	0.528	0.551
Within R^2	0.052	0.054	0.069

Notes: This table presents the first stage of panel fixed-effect 2SLS estimates, in which we use the distance from a large city of the linguistic area, i.e. a city in the top 5% of the population distribution, as an instrumental variable. The dependent variables are indicators of the presence of practical arithmetic manuals in the city in century $t-1$, with t ranging between the 14^{th} and 17^{th} centuries. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of century t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

B.3 Controlling for spatial correlation and spillover effects.

Spatial correlation. Table B6 shows the panel fixed-effect estimates with Conley standard errors, which control for potential spatial correlation in the data.

Table B6: Practical arithmetic manuals and the city population at the end of the following century using Conley standard errors (50km).

Dependent Variable:	log(End-of-century population$_t$)		
	(1)	(2)	(3)
log(Nb. of manuals$_{t-1}$+1)	0.082**		
	(0.035)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.112***	
		(0.041)	
Manuals dummy$_{t-1}$			0.085
			(0.055)
Printing press$_{t-1}$	0.036	0.032	0.043
	(0.045)	(0.045)	(0.046)
University	0.004	0.009	0.004
	(0.061)	(0.062)	(0.061)
Capital	0.278***	0.274***	0.276**
	(0.106)	(0.105)	(0.107)
log(Beginning-of-century population$_t$)	0.294***	0.291***	0.299***
	(0.041)	(0.040)	(0.041)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.895	0.895	0.895
Within R^2	0.113	0.116	0.109

Notes: This table presents panel fixed-effect OLS estimates of equation 2. The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of the century t, and we add city and century fixed effects. Conley standard errors (50 km) are reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Spillover effects. Table B7 reports the panel fixed-effect estimates, including interaction terms with the log-transformed number of practical arithmetic manuals in a radius that ranges from 20 km to 100 km in order to test for possible spillover effects. The absence of significant interactions at distances between 20 km and 80 km suggests that commercial knowledge did not provide local positive spillovers. In contrast, the negative and significant interaction term in the 100 km radius indicates a regional competition effect: the growth premium associated with a city's own manuals is attenuated when regional rivals adopt similar arithmetic techniques. This can be explained by the fact that trade between cities at relatively short distances typically involved low-added value goods, whereas high-value goods were traded at much greater distances, so that cities located within a 100 km radius would likely be competitors in long-distance trade routes (Cipolla 1956, 56).

Table B7: Spillover effects: Practical arithmetic manuals and the city population at the end of the following century.

Dependent Variable:	log(End-of-century population$_t$)				
	(1)	(2)	(3)	(4)	(5)
log(Nb. of manuals$_{t-1}$+1)	0.080**	0.075*	0.085*	0.129***	0.155***
	(0.036)	(0.041)	(0.046)	(0.047)	(0.048)
log(Manuals in 20km$_{t-1}$+1)	0.002				
	(0.051)				
log(Nb. of manuals+1) $\times$ log(Manuals in 20km+1)$_{t-1}$	0.104				
	(0.139)				
log(Manuals in 40km$_{t-1}$+1)		-0.014			
		(0.023)			
log(Nb. of manuals+1) $\times$ log(Manuals in 40km+1)$_{t-1}$		0.018			
		(0.044)			
log(Manuals in 60km$_{t-1}$+1)			-0.040**		
			(0.019)		
log(Nb. of manuals+1) $\times$ log(Manuals in 60km+1)$_{t-1}$			-0.0002		
			(0.034)		
log(Manuals in 80km$_{t-1}$+1)				-0.037**	
				(0.017)	
log(Nb. of manuals+1) $\times$ log(Manuals in 80km+1)$_{t-1}$				-0.036	
				(0.024)	
log(Manuals in 100km$_{t-1}$+1)					-0.017
					(0.015)
log(Nb. of manuals+1) $\times$ log(Manuals in 100km+1)$_{t-1}$					-0.048**
					(0.021)
Printing press$_{t-1}$	0.036	0.037	0.047	0.055	0.049
	(0.043)	(0.043)	(0.044)	(0.045)	(0.044)
University	0.004	0.005	-0.006	-0.017	-0.010
	(0.064)	(0.065)	(0.065)	(0.064)	(0.063)
Capital	0.278***	0.277***	0.287***	0.293***	0.291***
	(0.099)	(0.099)	(0.100)	(0.100)	(0.102)
log(Beginning-of-century population$_t$)	0.292***	0.293***	0.292***	0.286***	0.284***
	(0.038)	(0.037)	(0.038)	(0.039)	(0.039)
City FE	Yes	Yes	Yes	Yes	Yes
Century FE	Yes	Yes	Yes	Yes	Yes
Observations	1,799	1,799	1,799	1,799	1,799
R^2	0.895	0.895	0.896	0.896	0.896
Within R^2	0.114	0.114	0.117	0.123	0.122

Notes: This table presents panel fixed-effect OLS estimates of equation 2 with interaction terms between the number of practical arithmetic manuals in the city and the number of manuals in the cities within a radius (20 km, 40 km, 60 km, 80 km, or 100 km). The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Manuals* variable is an indicator of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of the century t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

B.4 Anticipation and placebo-treatment tests in the event studies.

Controlling for adopters' anticipation. Table B8 reports results of event study analysis described in previous sections when we relax the no-anticipation assumption and set anticipation equal to 1. In this way, we account for any early-stage shifts in the outcome variables that might precede the official adoption date. The procedure entails a loss of observations and a comparison with records at t-1. For these reasons, we observe an increase in confidence intervals in some event study ATTs. However, point estimates are in line with our main results, and overall ATTs are always significant. Importantly, the point estimate of the ATT of practical arithmetic manuals on the number of *Ars Mercatoria* texts increases when we relax the anticipation assumption, supporting the claim that practical arithmetic manuals are drivers for commercial texts.

Placebo-treatment tests. Table B9 summarizes the results of the event study analysis in which we randomize either the treatment periods or treated cities. Randomization is performed by preserving the temporal distribution of treatment. In the first placebo-treatment test, we randomize the treatment period across treated cities. In the second one, we randomly assign treatment to untreated cities. Placebo-treatment tests on the effects of practical arithmetic manuals on the number of notable merchants always fail the parallel-trend assumption because the pre-test p-value is zero.

Table B8: Event study results with anticipation equal to 1.

	log(End-of-century pop) (1)	log(Nb. of merchants+1) (2)	log(Nb. of AM texts+1) (3)
Pre-test p-value	0.11	0.23	0.20
Overall ATT	0.239*	0.055*	0.560*
	(0.120)	(0.025)	(0.089)
ATT t=-3	-0.109	-0.015	-0.014
	(0.165)	(0.021)	(0.022)
ATT t=-2	-0.012	0.013	-0.026
	(0.107)	(0.016)	(0.027)
ATT t=-1	-0.058	-0.020	0.083
	(0.083)	(0.025)	(0.047)
ATT t=0	0.183	0.054	0.660*
	(0.102)	(0.026)	(0.083)
ATT t=1	0.305	0.059	0.617*
	(0.136)	(0.033)	(0.101)
ATT t=2	0.162	0.039	0.500*
	(0.136)	(0.035)	(0.104)
ATT t=3	0.304	0.068	0.462*
	(0.161)	(0.033)	(0.119)

Notes: The table presents DiD coefficients estimated following equation 3 for European cities, with anticipation equal to 1. Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of the city population at the end of the century (specification 1), the logarithm of the number of 15-year-old notable merchants plus 1 in the 20-year interval (specification 2), and the logarithm of the number of *Ars Mercatoria* (AM) texts plus 1 in the 20-year interval (specification 3). Treatment occurs at time 0. Pre-treatment is conditional on the number of notable 15-year-olds (specification 2) or the number of printed books (specification 3) in the city. * indicates significant estimates based on a confidence interval of 95%. Standard errors are clustered at the city level.

Table B9: Event study results with random treatment.

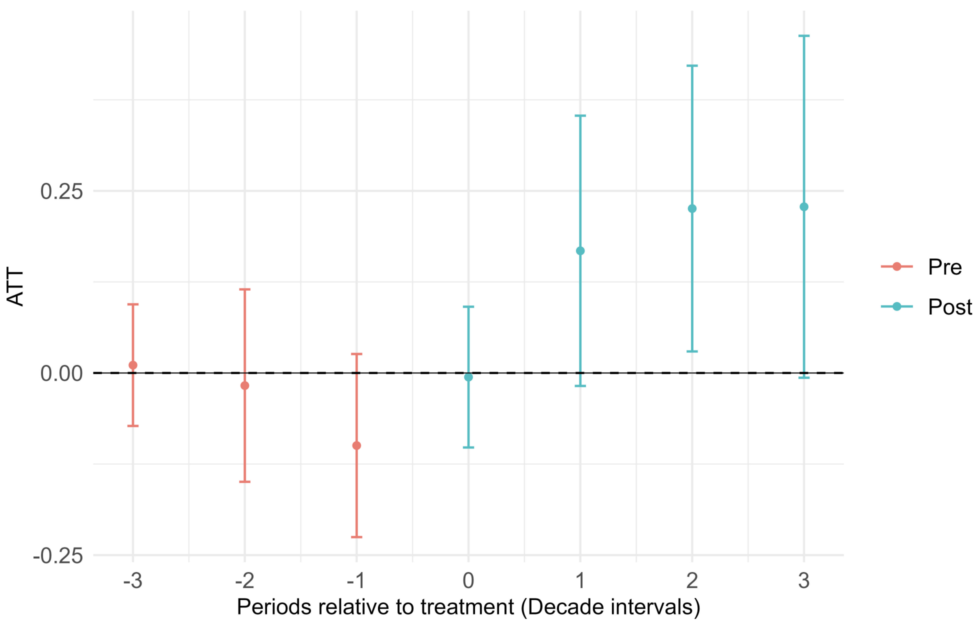
Random treatment period			
	log(End-of-century population) (1)	log(Nb. of merchants+1) (2)	log(Nb. of AM texts+1) (3)
Pre-test p-value	0.09	0.00	0.21
Overall ATT	0.150	0.009	0.088
	(0.097)	(0.028)	(0.086)
ATT t=-3	-0.131	0.022	0.046
	(0.163)	(0.025)	(0.050)
ATT t=-2	0.032	-0.005	-0.009
	(0.091)	(0.019)	(0.053)
ATT t=-1	-0.094	-0.012	0.131
	(0.085)	(0.024)	(0.077)
ATT t=0	0.164	-0.011	0.012
	(0.076)	(0.028)	(0.074)
ATT t=1	0.215	0.010	0.061
	(0.095)	(0.025)	(0.092)
ATT t=2	0.070	0.012	0.105
	(0.108)	(0.035)	(0.105)
ATT t=3	0.150	0.024	0.175
	(0.141)	(0.041)	(0.126)
Random treated cities			
	log(End-of-century population) (1)	log(Nb. of merchants+1) (2)	log(Nb. of AM texts+1) (3)
Pre-test p-value	0.22	0.00	0.85
Overall ATT	-0.012	-0.010	-0.022
	(0.113)	(0.011)	(0.015)
ATT t=-3	0.269	-0.001	-0.004
	(0.335)	(0.001)	(0.003)
ATT t=-2	0.137	0.006	-0.000
	(0.221)	(0.010)	(0.015)
ATT t=-1	-0.030	-0.002	-0.008
	(0.137)	(0.011)	(0.013)
ATT t=0	0.051	-0.003	-0.018
	(0.122)	(0.010)	(0.014)
ATT t=1	-0.013	-0.012	-0.028
	(0.127)	(0.011)	(0.015)
ATT t=2	0.021	-0.013	-0.007
	(0.132)	(0.010)	(0.020)
ATT t=3	-0.107	-0.013	-0.036
	(0.136)	(0.010)	(0.024)

Notes: The table presents DiD coefficients estimated following equation 3 for European cities in the case of a random treatment period (top panel) or random treated cities (bottom panel). Control cities are those that have not yet been treated. The dependent variable is the logarithm of the city population at the end of the century (specification 1), the logarithm of the number of 15-year-old notable merchants plus 1 in the 20-year interval (specification 2), and the logarithm of the number of *Ars Mercatoria* (AM) texts plus 1 in the 20-year interval (specification 3). Treatment occurs at time 0. Pre-treatment is conditional on the number of notable 15-year-olds (specification 2) or the number of printed books (specification 3) in the city. Confidence intervals are 95%. Standard errors are clustered at the city level.

B.5 Alternative data, sample selections, alternative variables, and alternative controls.

Real wages of skilled workers in selected cities. We study the effect of the introduction of practical arithmetic manuals on cities' average real wage of skilled workers at the decade level. Data on average real wages are retrieved from Serafinelli and Tabellini (2022), based on evidence gathered by Allen (2001), and are available for 17 cities in our sample across 8 European countries. Data on skilled wages, even though they are available for a smaller sample of cities, make it possible to investigate the relationship between practical arithmetic manuals and a more direct measure of economic development because practitioners trained in practical arithmetic certainly feature among skilled workers, and were active in a broad range of numerically-intensive sectors, such as commerce, finance, surveying, and the mechanical arts. Figure B2 shows the event study performed on this sample. We consider not-yet-treated cities as a control group. The average treatment effect on the average real wages of skilled workers in treated cities is positive, statistically significant, and equal to 0.154 (Std. Error: 0.072).

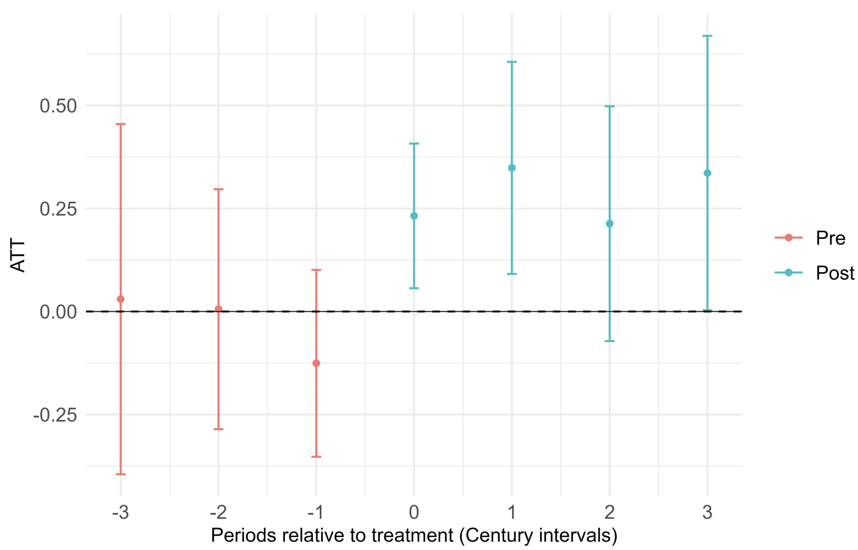
Figure B2: Event study for the average real wage of skilled workers.



Notes: The figure shows DiD coefficients estimated following equation 3 for European cities. Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of the average real wage of skilled workers in a decade. Treatment occurs at time 0. Confidence intervals are 95%. Standard errors are clustered at the city level.

Population in European cities based on Buringh's data. Tables B10 and B11 report the panel fixed-effect results derived by a replication of the previous analyses on a sub-sample of data, i.e., considering only observations in the Buringh (2021)'s dataset of European cities' population. Figures B3, B4, and B5 show the event study performed on this sample. The overall ATT for city population at the end of the century is positive, statistically significant, and equal to 0.283 (Std. Error: 0.090), corresponding to a 32.71% increase in the city population during the three centuries following the first adoption of practical arithmetic manuals. Finally, the overall ATTs for the number of notable merchants and the number of *Ars Mercatoria* texts are, respectively, 0.077 (Std. Error: 0.026) and 0.422 (Std. Error: 0.079).

Figure B3: Event study on the city's population at the end of the century, as estimated by Buringh.



Notes: The figure shows DiD coefficients estimated following equation 3 for European cities included in Buringh (2021). Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of city population at the end of the century. Treatment occurs at time 0. P-value for pre-test of parallel trends assumption (Ward test): 0.20. Confidence intervals are 95%. Standard errors are clustered at the city level.

Table B10: Practical arithmetic manuals and Buringh's city population at the end of the following century.

Dependent Variable:	log(End-of-century population$_t$)		
	(1)	(2)	(3)
log(Nb. of manuals$_{t-1}$+1)	0.071*		
	(0.037)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.100**	
		(0.043)	
Manuals dummy$_{t-1}$			0.065
			(0.061)
Printing press$_{t-1}$	0.016	0.011	0.024
	(0.050)	(0.050)	(0.051)
University	-0.019	-0.016	-0.019
	(0.073)	(0.074)	(0.072)
Capital	0.392***	0.388***	0.391***
	(0.110)	(0.108)	(0.111)
log(Beginning-of-century population$_t$)	0.309***	0.306***	0.315***
	(0.042)	(0.041)	(0.042)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,538	1,538	1,538
R^2	0.905	0.905	0.905
Within R^2	0.120	0.123	0.117

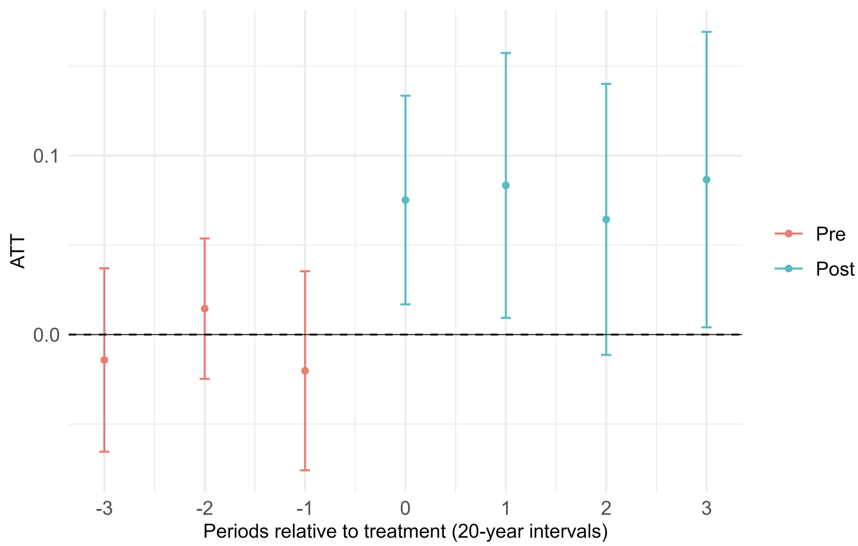
Notes: This table presents panel fixed-effect OLS estimates of equation 2 for European cities included in Buringh (2021). The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The $Manuals$ variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of the century t, and city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B11: Practical arithmetic manuals, the 15-year-old notable merchants, and *Ars Mercatoria* texts in Buringh's cities.

Dependent Variables:	log(Nb. of merchants$_{t+1}$+1)		log(Nb. of AM texts$_{t+1}$+1)	
	(1)	(2)	(3)	(4)
log(Nb. of manuals$_t$+1)	0.064***		0.205***	
	(0.018)		(0.031)	
Manuals dummy$_t$		0.066***		0.130***
		(0.022)		(0.043)
Printing press$_t$	0.001	0.002		
	(0.004)	(0.004)		
University	-0.015	-0.015	0.025	0.025
	(0.009)	(0.010)	(0.022)	(0.024)
Capital	-0.006	-0.007	0.036	0.022
	(0.009)	(0.009)	(0.033)	(0.033)
log(Nb. of merchants$_t$+1)	0.115**	0.127**		
	(0.056)	(0.060)		
log(Nb. of notable people$_t$+1)	0.016***	0.018***		
	(0.004)	(0.005)		
log(Nb. of printed books$_t$+1)			0.036***	0.045***
			(0.005)	(0.005)
log(Nb. of *Ars Mercatoria* texts$_t$+1)			0.347***	0.384***
			(0.031)	(0.032)
City FE	Yes	Yes	Yes	Yes
20y interval FE	Yes	Yes	Yes	Yes
Observations	10,990	10,990	8,635	8,635
R^2	0.352	0.347	0.630	0.623
Within R^2	0.051	0.043	0.247	0.233

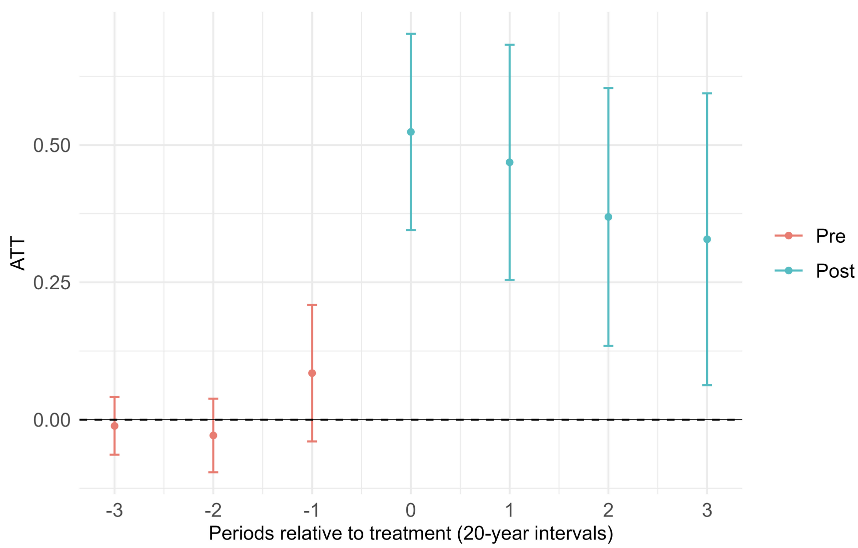
Notes: This table presents panel fixed-effect OLS estimates of equation 4 (specifications 1–2) and 5 (specifications 3–4) for European cities included in Buringh (2021). The dependent variables are the logarithm of the number of 15-year-old notable merchants + 1 at time $t+1$ and the logarithm of the number of *Ars Mercatoria* (AM) texts + 1 at time $t+1$. t indicates the 20-year intervals in the 14th–16th centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city at $t-1$. Control variables include the presence of printing presses, a university, a state capital, and the number of printed books. We also control for the number of 15-year-old notable merchants, notable people, or *Ars Mercatoria* texts at time t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Figure B4: Event study on the city's 15-year-old notable merchants in 20 years in Buringh's cities.



Notes: The figure shows DiD coefficients estimated following equation 3 for European cities included in Buringh (2021). Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of the number of 15-year-old notable merchants plus 1 in the 20-year interval. Treatment occurs at time 0. Pre-treatment is conditional on the number of notable 15-year-olds in the city. P-value for pre-test of parallel trends assumption (Ward test): 0.39. Confidence intervals are 95%. Standard errors are clustered at the city level.

Figure B5: Event study on the number of *Ars Mercatoria* texts printed in 20 years in Buringh's cities.



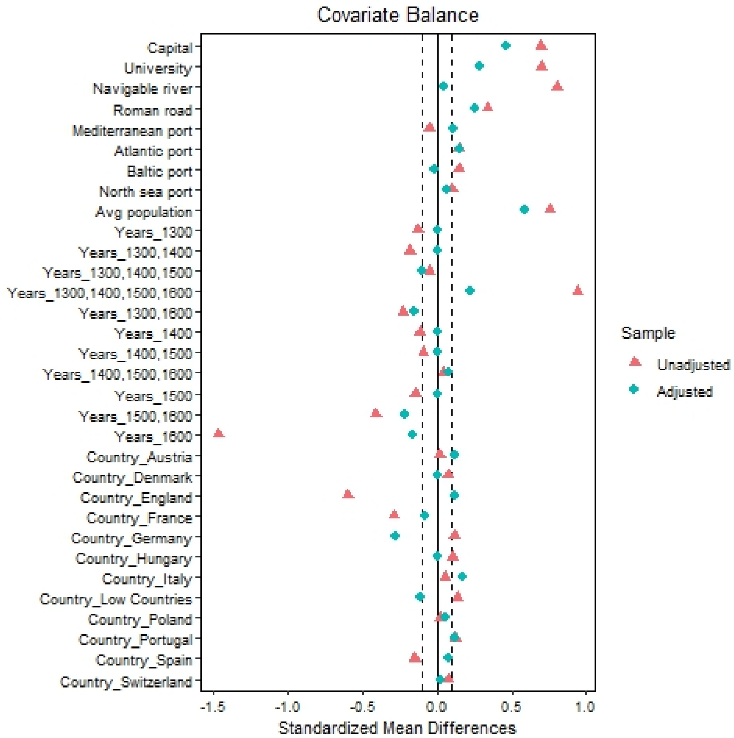
Notes: The figure shows DiD coefficients estimated following equation 3 for European cities included in Buringh (2021). Treatment is defined as the first appearance of a practical arithmetic manual in the city. Control cities are those that have not yet been treated. The dependent variable is the logarithm of the number of *Ars Mercatoria* texts plus 1 in the 20-year interval. Treatment occurs at time 0. Pre-treatment is conditional on the number of printed books in the city. P-value for pre-test of parallel trends assumption (Ward test): 0.23. Confidence intervals are 95%. Standard errors are clustered at the city level.

Matched sample. To identify pairs of cities with similar features, we computed a propensity score, estimated with logistic regression, in terms of observables on both century- and 20-year-interval data. Cities are matched using nearest-neighbor matching without replacement based on the propensity score difference. Figure B6 shows the difference between the characteristics of cities with and without manuals before and after the nearest neighbor matching based on propensity score difference.

Results on city population are reported in table B12 (specifications 1–3). The estimated association between the presence of practical arithmetic manuals and urban growth in the following century is similar to the one obtained in table 2. However, by reducing the sample of cities and balancing places with and without texts, in this set of estimates, the coefficient associated with the dummy variable is positive and significant. This is a useful complement to the results obtained with the number of manuals in interpreting the economic importance of practical mathematics. Results are confirmed when considering alternative matching strategies, such as nearest neighbor matching based on the Mahalanobis distance (specifications 4–6) and optimal pair matching (specifications 7–9).[19] Results on the number of notable merchants and the number of *Ars Mercatoria* texts are reported in table B13, confirming findings in tables 3 and B1.

19. It is interesting to notice that, in these empirical designs, there is no effect associated with the presence of the printing press.

Figure B6: Distance between cities with and without practical arithmetic manuals before and after matching on century and 20-year interval data.



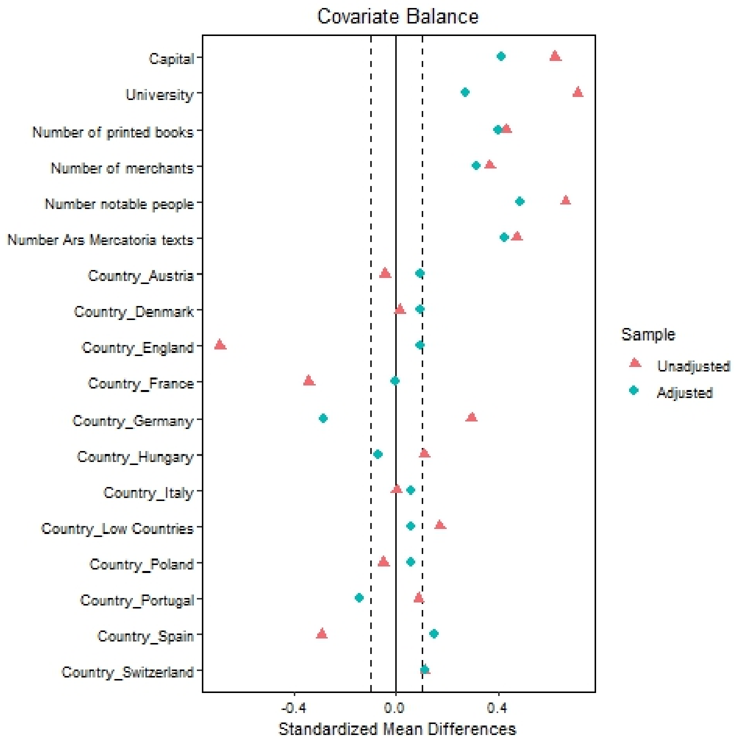


Table B12: Matching estimates. Practical arithmetic manuals and the city population at the end of the following century.

Dependent Variable:	log(End-of-century population$_t$)								
	Nearest-neighbor			Mahalanobis distance			Optimal pair		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
log(Nb. of manuals$_{t-1}$+1)	0.091***			0.095***			0.103***		
	(0.035)			(0.035)			(0.035)		
log(Nb. of man 2^{nd} half-cent$_{t-1}$+1)		0.120***			0.123***			0.133***	
		(0.040)			(0.040)			(0.040)	
Manuals dummy$_{t-1}$			0.104*			0.110**			0.122**
			(0.053)			(0.054)			(0.053)
Printing press$_{t-1}$	0.042	0.034	0.042	0.064	0.057	0.065	0.042	0.034	0.042
	(0.063)	(0.063)	(0.064)	(0.060)	(0.060)	(0.061)	(0.063)	(0.063)	(0.064)
University	0.033	0.039	0.029	0.048	0.054	0.045	0.042	0.048	0.038
	(0.063)	(0.063)	(0.062)	(0.067)	(0.067)	(0.066)	(0.064)	(0.064)	(0.063)
Capital	0.310***	0.307***	0.309***	0.353***	0.349***	0.351***	0.312***	0.309***	0.312***
	(0.087)	(0.086)	(0.088)	(0.090)	(0.089)	(0.091)	(0.088)	(0.087)	(0.089)
log(Beg-of-century population$_t$)	0.383***	0.378***	0.396***	0.381***	0.376***	0.395***	0.370***	0.365***	0.384***
	(0.060)	(0.058)	(0.060)	(0.061)	(0.059)	(0.061)	(0.060)	(0.058)	(0.060)
City FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Century FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	641	641	641	644	644	644	653	653	653
R^2	0.890	0.891	0.889	0.896	0.897	0.895	0.887	0.888	0.886
Within R^2	0.221	0.225	0.212	0.236	0.241	0.227	0.210	0.215	0.200

Notes: This table presents panel fixed-effect OLS estimates of Equation 2 on matched cities. Matching is performed using nearest-neighbor matching based on propensity score differences across cities (specifications 1–3), Mahalanobis distance (specifications 4–6), or optimal pair matching (specifications 7–9). The dependent variable is the logarithm of the population level at the end of century t. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the city in the century $t-1$, the university, and the state capital. We also control for the logarithm of the city population at the beginning of the century t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B13: Matching estimates. Practical arithmetic manuals, the 15-year-old notable merchants, and *Ars Mercatoria* texts.

Dependent Variables:	log(Nb. of merchants$_{t+1}$+1)		log(Nb. of AM texts$_{t+1}$+1)	
	(1)	(2)	(3)	(4)
log(Nb. of manuals$_t$+1)	0.055***		0.192***	
	(0.016)		(0.032)	
Manuals dummy$_t$		0.054***		0.100**
		(0.020)		(0.043)
University	-0.019	-0.019	-0.004	-0.001
	(0.012)	(0.013)	(0.032)	(0.036)
Capital	-0.007	-0.009	0.054	0.033
	(0.010)	(0.010)	(0.044)	(0.044)
printing_press	0.0006	0.001		
	(0.008)	(0.008)		
log(Nb. of merchants$_t$+1)	0.150**	0.163**		
	(0.063)	(0.067)		
log(Nb. of notable people$_t$+1)	0.029***	0.031***		
	(0.009)	(0.009)		
log(Nb. of printed books$_t$+1)			0.042***	0.054***
			(0.007)	(0.008)
log(Nb. of *Ars Mercatoria* texts$_t$+1)			0.332***	0.374***
			(0.034)	(0.035)
City FE	Yes	Yes	Yes	Yes
20y interval FE	Yes	Yes	Yes	Yes
Observations	2,884	2,884	2,266	2,266
R^2	0.390	0.385	0.633	0.626
Within R^2	0.067	0.060	0.231	0.216

Notes: This table presents panel fixed-effect OLS estimates of equation 4 (specifications 1–2) and 5 (specifications 3–4) on matched cities. Matching is performed using nearest-neighbor matching based on propensity score differences across cities. The dependent variables are the logarithm of the number of 15-year-old notable merchants + 1 at time $t+1$ and the logarithm of the number of *Ars Mercatoria* (AM) texts + 1 at time $t+1$. t indicates the 20-year intervals in the 14th–16th centuries. The $Manuals$ variables are indicators of the presence of practical arithmetic manuals in the city at $t-1$. Control variables include the presence of printing presses, a university, a state capital, and the number of printed books. We also control for the number of 15-year-old notable merchants, notable people, or *Ars Mercatoria* texts at time t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Balanced panel in city-century regressions. We observe population data for the entire period of analysis (14th–17th centuries) in 215 cities. We replicate panel fixed-effect estimates by considering those cities exclusively to obtain a balanced panel. Results, reported in table B14, show that the association between the presence of public arithmetic manuals and city growth is stronger for this subsample, and the coefficient associated with the manual dummy variable is positive and significant.

Table B14: Practical arithmetic manuals and city population at the end of the following century - Balanced panel.

Dependent Variable:	log(End-of-century population$_t$)		
log(Nb. of manuals$_{t-1}$+1)	0.134***		
	(0.042)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.185***	
		(0.046)	
Manuals dummy$_{t-1}$			0.139**
			(0.062)
Printing press$_{t-1}$	-0.016	-0.024	-0.004
	(0.053)	(0.053)	(0.055)
University	0.006	0.013	0.007
	(0.073)	(0.073)	(0.072)
Capital	0.335***	0.335***	0.331***
	(0.098)	(0.096)	(0.101)
log(Beginning-of-century population$_t$)	0.323***	0.318***	0.335***
	(0.045)	(0.044)	(0.046)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	860	860	860
R^2	0.863	0.864	0.861
Within R^2	0.172	0.179	0.161

Notes: This table presents balanced panel fixed-effect OLS estimates of equation 2 for European cities observed throughout the entire period of analysis. The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of the century t, and city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Placebo test: Ptolemaic astronomy manuscripts It can be argued that the effects attributed to practical arithmetic manuals are part of a more general effect of mathematical knowledge that could have diffused via alternative channels. To show that this is not the case, and therefore rule out this alternative explanation, we design a placebo test in which we analyze the influence on urban growth of arithmetical knowledge diffused in universities and monasteries. The rationale of this test is to explore the importance of economic performance, not just of the diffusion of knowledge in general, but also, and more specifically, of its social circulation and its application to economic problems. To do so, we focus on the circulation of Indo-Arabic numerals in theoretical—as opposed to practical—sources. While the main evidence of this paper is provided by data on the diffusion of vernacular manuals that circulated among practitioners, in this section, we focus on mathematical texts that circulated among scholars.

In fact, the vernacular tradition of practical arithmetic was not the only tradition spreading Indo-Arabic numerals in Europe. The initial circulation of the positional numeral system in Europe started in the 12th-century *al-Andalus* with the first Latin translations of Arabic mathematical works (Folkerts and Kunitzsch 1997; Folkerts 2003). These translations were carried out by European scholars proficient in both Arabic and Latin, and their subsequent copies constitute the tradition of the so-called Latin *algorismi*. These texts are brief mathematical primers that were used to show how to use Indo-Arabic numerals to people who could access a Latin education. While they include fundamentally the same mathematical knowledge as vernacular manuals, Latin *algorismi* are theoretical texts, and generally do not deal with the practical applications of mathematics. As such, they were quickly adopted in the main centers of European learning—i.e., monasteries and universities—as they were relevant for scholars interested in advanced calculations. These were needed primarily to solve problems arising in astronomy, such as accurate calendrical calculations. As a consequence, Latin *algorismi* are often found in manuscripts which also include texts on astronomy (Allard 1991; Nothaft 2014). As they were primarily circulating in monasteries and universities, the geographical spread of Latin *algorismi* was different from that of practical arithmetic texts. While these texts include the same mathematical theory as practical arithmetic manuals—the same 'technology'—they circulated in different social and institutional settings, and their mathematical knowledge was applied to solve different problems. As observed in the incipit of a Florentine practical arithmetic manual written around 1457, the learned mathematics spread by Latin *algorismi* was of little use to economic agents active on the market.[20] We, therefore, use evidence on the diffusion of this Latin tradition to investigate the impact of the same factor as that of practical arithmetic (i.e. Indo-Arabic numerals) applied in different social and institutional settings and to the solution of different problems. Specifically, we replicate the panel fixed-effect analysis by considering the diffusion in European cities in the 13th–16th centuries of Latin astronomical texts that use advanced arithmetic, relying on data on the Latin tradition of Ptolemaic texts.[21]

As in the case of Latin *algorismi*, Europe came in contact with Ptolemy's works thanks to the first translations of his texts from Arabic into Latin. For example, one of Ptolemy's main works—the *Almagest*—was rediscovered by Western Europe thanks to the translations from Arabic to Latin carried out by Gerard of Cremona in late 12th-century *al-Andalus* (Kunitzsch 1990). Ptolemy's works were among the most advanced astronomical texts of the time and required advanced mathematical skills to be understood.

We retrieve information on Ptolemy's manuscripts written in Latin from the data collected by the *Ptolemaeus Arabus et Latinus* (PAL) project, which provides a reconstruction of the entire Latin *corpus Ptolemaicus*, i.e. of all Latin manuscripts which include works by or attributed to Ptolemy (Folkerts and Lorch 2000; Kunitzsch 2004).[22] The database provides

20. This is Francesco di Carlo de Macigni's *Libretto alla praticha della merchatantia*, preserved in Florence, Biblioteca Mediceo-Laurenziana, ms. Ash. 352. In the incipit of the text, Francesco remarked: "uno che voglia essere marchatante non ha bisogno d'inparare a misurare le stelle e pianeti e 'l chorso de' pianeti e movimenti de' cieli, ma solo ha bisogno d'intendere la marchatantia e lle ragioni ch'elli adoperano in essa merchatantia", i.e. "One who wants to become a merchant does not need to learn how to measure stars and planets and their course or that of the skies, but only needs to know well the trade of merchants and the problems which are needed in their business".

21. There is no complete survey of all surviving Latin *algorismi*, but evidence of the Latin Ptolemaic tradition is a reliable proxy of the circulation of Arabic arithmetic in the centers of learning in Europe.

22. Data are available here: PAL: Latin Manuscripts.

Table B15: Ptolemy's manuscripts and city population at the end of the following century.

	log(End-of-century population$_t$)		
	(1)	(2)	(3)
log(Nb. of Ptolemy's man$_{t-1}$+1)	-0.082		
	(0.069)		
log(Nb. of Ptolemy's man 2^{nd} half-century$_{t-1}$+1)		-0.088	
		(0.070)	
Ptolemy's manuscripts dummy$_{t-1}$			-0.086
			(0.066)
Printing press$_{t-1}$	0.066	0.063	0.065
	(0.044)	(0.044)	(0.044)
University	-0.004	-0.0004	-0.004
	(0.063)	(0.063)	(0.063)
Capital	0.260**	0.270***	0.259**
	(0.102)	(0.101)	(0.102)
log(Beginning-of-century population$_t$)	0.303***	0.303***	0.304***
	(0.039)	(0.039)	(0.039)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.895	0.895	0.895
Within R^2	0.109	0.109	0.108

Notes: This table presents panel fixed-effect OLS estimates on the placebo dataset. The dependent variable is the logarithm of the population level at the end of the century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Ptolemy's manuscripts* variables are indicators of the presence of Ptolemy's manuscripts in the city in century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of century t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

information about the estimated date, origin, and provenance of 681 manuscripts belonging to the Ptolemaic tradition. By exploiting the information on the origin of manuscripts provided by the PAL project, we can locate Ptolemaic manuscripts in 57 cities (10 countries) of our database. Panel fixed-effect estimates are reported in table B15 and show no significant effects of the presence of Ptolemaic manuscripts on urban growth. This rules out the academic channel as a possible alternative mechanism of growth.[23]

23. We do not report the event study analysis, which results in a non-significant ATT, since the parallel-trend assumption is not satisfied in this setting.

Heterogeneous effects by city size. We define small cities as cities with up to 3,000 inhabitants at the beginning of the century.[24] Table B16 presents panel fixed-effect estimates in which we interact the *Manuals* variables with a dummy variable indicating small cities. The interaction term is positive and significant for all possible indicators of the presence of manuals, while the coefficients associated with our main dependent variables remain qualitatively unchanged.

24. 11% of city-century observation has 3,000 inhabitants or fewer. Results are robust for different thresholds on the population distribution.

Table B16: Practical arithmetic manuals and the city population at the end of the following century in small cities (up to 3,000 inhabitants).

Dependent Variable:	log(End-of-century population$_t$)		
	(1)	(2)	(3)
log(Nb. of manuals$_{t-1}$+1)	0.080**		
	(0.035)		
log(Nb. of manuals$_{t-1}$+1) × Small city	0.618***		
	(0.105)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.109***	
		(0.041)	
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1) × Small city		0.590***	
		(0.107)	
Manuals dummy$_{t-1}$			0.076
			(0.055)
Manuals dummy$_{t-1}$ × Small city			0.404***
			(0.090)
Printing press$_{t-1}$	0.036	0.032	0.043
	(0.044)	(0.044)	(0.045)
University	0.003	0.008	0.002
	(0.064)	(0.064)	(0.064)
Capital	0.278***	0.274***	0.275***
	(0.099)	(0.098)	(0.100)
Small city	0.010	0.007	0.022
	(0.064)	(0.064)	(0.064)
log(Beginning-of-century population$_t$)	0.297***	0.294***	0.307***
	(0.042)	(0.041)	(0.042)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.895	0.896	0.895
Within R^2	0.114	0.117	0.110

Notes: This table presents panel fixed-effect OLS estimates of Equation 2 in which we include an interaction term between the variable *Manuals* and a dummy variable that captures the small size of the city (cities up to 3,000 inhabitants). The dependent variable is the logarithm of the population level at the end of the century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of century t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Alternative controls. Table B17 reports the results of the pooled OLS estimates with the city's longitude, its latitude, and their interaction as control variables.

Tables B18 and B19 show, respectively, the results of panel fixed-effect estimates with additional country-x-century fixed effects and Black Death controls. Data on the cities affected by the Black Death in the 14th century have been retrieved by Christakos et al. (2005), Gingerich and Vogler (2021), Benedictow (2021), Sandri and Ginatempo (1990), Byrne (2012), Alfani and Ammannati (2017), and Herlihy (1997). In total, historical sources document that 112 out of 282 cities have been impacted by the plague.

Tables B20 and B21 present results on the association between practical arithmetic manuals and the number of 15-year-old notable merchants or the number of *Ars Mercatoria* texts in the following 20-years when controlling for the growth of notable people between the two periods and the estimated population growth (computed as the century population growth divided by 5, when available).

Table B17: Alternative controls: Practical arithmetic manuals and the city population at the end of the following century.

	log(End-of-century population$_t$)		
	(1)	(2)	(3)
log(Nb. of manuals$_{t-1}$+1)	0.109***		
	(0.029)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.156***	
		(0.032)	
Manuals dummy$_{t-1}$			0.118**
			(0.053)
Printing press$_{t-1}$	0.162***	0.159***	0.166***
	(0.033)	(0.033)	(0.034)
University	0.078*	0.081**	0.082**
	(0.041)	(0.041)	(0.041)
Capital	0.377***	0.373***	0.388***
	(0.065)	(0.065)	(0.066)
log(Beginning-of-century population$_t$)	0.714***	0.711***	0.721***
	(0.018)	(0.018)	(0.018)
Longitude	0.091***	0.091***	0.092***
	(0.016)	(0.016)	(0.016)
Latitude	0.010**	0.010**	0.010**
	(0.004)	(0.004)	(0.004)
Longitude × Latitude	-0.002***	-0.002***	-0.002***
	(0.0004)	(0.0004)	(0.0004)
Geographical controls	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.727	0.728	0.725
Within R^2	0.718	0.719	0.717

Notes: This table presents pooled OLS estimates of equation 1. The dependent variable is the logarithm of the population level at the end of century t. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, state capital, and geographical controls (Roman roads, navigable rivers, and seaport). We control for the log city population, longitude, and latitude. We add century fixed effects. Heteroskedasticity-robust standard errors are clustered at the country level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B18: Practical arithmetic manuals and city population at the end of the following century with country-x-century fixed effects.

Dependent Variable:	log(End-of-century population$_t$)		
log(Nb. of manuals$_{t-1}$+1)	0.079***		
	(0.030)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.103***	
		(0.034)	
Manuals dummy$_{t-1}$			0.082
			(0.051)
Printing press$_{t-1}$	0.005	0.002	0.011
	(0.043)	(0.043)	(0.044)
University	0.005	0.009	0.004
	(0.064)	(0.064)	(0.063)
Capital	0.233**	0.230**	0.231**
	(0.097)	(0.096)	(0.098)
log(Beginning-of-century population$_t$)	0.281***	0.279***	0.287***
	(0.037)	(0.037)	(0.038)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Country-Century FE	Yes	Yes	Yes
Fit statistics			
Observations	1,799	1,799	1,799
R^2	0.911	0.911	0.911
Within R^2	0.105	0.107	0.101

Notes: This table presents panel fixed-effect OLS estimates of equation 2 for European cities. The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, and state capital. We also control for the logarithm of the city population at the beginning of the century t, and city, century, and country x century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B19: Practical arithmetic manuals and city population at the end of the following century with Black Death controls.

Dependent Variable:	log(End-of-century population$_t$)		
log(Nb. of manuals$_{t-1}$+1)	0.085**		
	(0.036)		
log(Nb. of manuals 2^{nd} half-century$_{t-1}$+1)		0.114***	
		(0.041)	
Manuals dummy$_{t-1}$			0.088
			(0.055)
Printing press$_{t-1}$	0.038	0.034	0.045
	(0.043)	(0.044)	(0.044)
University	0.009	0.013	0.007
	(0.065)	(0.065)	(0.064)
Capital	0.277***	0.273***	0.275***
	(0.100)	(0.098)	(0.101)
Black Death	0.039	0.040	0.027
	(0.072)	(0.072)	(0.073)
log(Beginning-of-century population$_t$)	0.291***	0.289***	0.298***
	(0.038)	(0.038)	(0.039)
City FE	Yes	Yes	Yes
Century FE	Yes	Yes	Yes
Observations	1,799	1,799	1,799
R^2	0.895	0.896	0.895
Within R^2	0.114	0.116	0.109

Notes: This table presents panel fixed-effect OLS estimates of equation 2 for European cities. The dependent variable is the logarithm of the population level at the end of century t, with t ranging between the 14^{th} and 17^{th} centuries. The $Manuals$ variables are indicators of the presence of practical arithmetic manuals in the city in the century $t-1$. Control variables include the presence of printing presses in the century $t-1$, university, state capital, and Black Death. We also control for the logarithm of the city population at the beginning of the century t, and city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B20: Alternative controls: Practical arithmetic manuals and the number of 15-year-old notable merchants in the following 20 years.

Dependent Variables:	log(Nb. of merchants$_{t+1}$+1)				Merchants dummy$_{t+1}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(Nb. of manuals$_t$+1)	0.068***	0.069***			0.070***	0.067***		
	(0.019)	(0.022)			(0.020)	(0.024)		
Manuals dummy$_t$			0.071***	0.068***			0.076***	0.070***
			(0.023)	(0.024)			(0.025)	(0.026)
Printing press$_t$	0.008**	0.004	0.009**	0.005	0.013***	0.008	0.014***	0.009
	(0.004)	(0.007)	(0.004)	(0.007)	(0.005)	(0.008)	(0.005)	(0.008)
University	-0.014	-0.016	-0.014	-0.014	-0.024***	-0.031**	-0.024***	-0.030**
	(0.009)	(0.014)	(0.009)	(0.015)	(0.009)	(0.014)	(0.009)	(0.014)
Capital	-0.002	0.015	-0.003	0.013	-0.002	0.023	-0.002	0.021
	(0.009)	(0.010)	(0.009)	(0.011)	(0.012)	(0.015)	(0.013)	(0.015)
log(Nb. of merchants$_t$+1)	0.125**	0.101	0.140**	0.117	0.057	0.016	0.071	0.030
	(0.058)	(0.066)	(0.063)	(0.072)	(0.048)	(0.054)	(0.052)	(0.058)
Notable people growth$_{t,t+1}$	0.009***		0.009***		0.011***		0.011***	
	(0.002)		(0.002)		(0.002)		(0.002)	
Population growth (estimated) $_{t,t+1}$		0.004		0.007		-0.001		0.002
		(0.035)		(0.036)		(0.036)		(0.037)
City FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
20y interval FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11,578	5,053	11,578	5,053	11,578	5,053	11,578	5,053
R^2	0.348	0.397	0.342	0.392	0.281	0.340	0.277	0.337
Within R^2	0.047	0.037	0.038	0.028	0.026	0.017	0.020	0.012

Notes: This table presents panel fixed-effect OLS estimates of equation 4. The dependent variables are the logarithm of the number of 15-year-old notable merchants + 1 at time $t+1$ and a dummy variable equal to one if there is at least one 15-year-old notable merchant at time $t+1$. t indicates the 20-year intervals in the 14th–16th centuries. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city at $t-1$. Control variables include the presence of printing presses, a university, and a state capital. We also control for the number of 15-year-old notable merchants and the growth of notable people or the estimated city growth at time t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

Table B21: Alternative controls: Practical arithmetic manuals and the number of *Ars Mercatoria* texts in the following 20 years.

Dependent Variables:	log(Nb. of AM texts$_{t+1}$+1)				AM text dummy$_{t+1}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(Nb. of manuals$_t$+1)	0.212***	0.219***			0.070***	0.082***		
	(0.033)	(0.033)			(0.022)	(0.024)		
Manuals dummy$_t$			0.142***	0.168***			0.097***	0.114***
			(0.044)	(0.048)			(0.030)	(0.033)
University	0.030	0.056	0.030	0.068	0.050*	0.088**	0.050*	0.091**
	(0.025)	(0.043)	(0.028)	(0.047)	(0.026)	(0.044)	(0.026)	(0.045)
Capital	0.054	0.053	0.037	0.029	0.014	0.001	0.011	-0.005
	(0.042)	(0.061)	(0.041)	(0.061)	(0.029)	(0.043)	(0.029)	(0.043)
log(Nb. of printed books$_t$+1)	0.026***	0.031***	0.035***	0.041***	0.024***	0.028***	0.024***	0.029***
	(0.005)	(0.006)	(0.005)	(0.006)	(0.004)	(0.005)	(0.004)	(0.005)
log(*Ars Mercatoria* books$_t$+1)	0.303***	0.271***	0.341***	0.307***	0.136***	0.107***	0.141***	0.113***
	(0.034)	(0.036)	(0.034)	(0.035)	(0.019)	(0.020)	(0.019)	(0.020)
Notable people growth$_{t,t+1}$	0.043***		0.044***		0.030***		0.031***	
	(0.008)		(0.009)		(0.005)		(0.005)	
Population growth (estimated) $_{t,t+1}$		-0.036		-0.039		0.0002		-0.0007
		(0.061)		(0.065)		(0.045)		(0.044)
City FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
20y interval FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	8,270	5,227	8,270	5,227	8,270	5,227	8,270	5,227
R^2	0.647	0.654	0.640	0.648	0.501	0.513	0.501	0.513
Within R^2	0.206	0.172	0.190	0.157	0.096	0.077	0.096	0.077

Notes: This table presents panel fixed-effect OLS estimates of equation 5. The dependent variables are the logarithm of the number of *Ars Mercatoria* (AM) texts + 1 at time $t+1$ and a dummy variable equal to one if there is at least one *Ars Mercatoria* text at time $t+1$. t indicates the 20-year intervals after the emergence of the printing press. The *Manuals* variables are indicators of the presence of practical arithmetic manuals in the city at $t-1$. Control variables include the presence of a university or a state capital. We also control for the number of printed books at time t, and we add city and century fixed effects. Heteroskedasticity-robust standard errors are clustered at the city level and reported in parentheses. Significance level: *p<0.1; **p<0.05; ***p<0.01

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